#34 $\sum_{n=1}^{\infty} n \tan \frac{1}{n}$

§10.3 questions

TEST FOR DIVERGENCE, FIRST.

$$\int_1^{\infty} x \tan\left(\frac{1}{x}\right) dx$$

$$\lim_{n\to\infty} n \tan\left(\frac{1}{n}\right) \quad 1^{st}$$

$$\lim_{n\to\infty} \frac{\tan\left(\frac{1}{n}\right)}{\frac{1}{n}} \overset{L'H}{=} \lim_{n\to\infty} \frac{\sec^2\left(\frac{1}{n}\right)\cdot -\frac{1}{n^2}}{-\frac{1}{n^2}} = \lim_{n\to\infty} \sec^2\left(\frac{1}{n}\right) = 1 \neq 0$$

Fails test for divergence, i.e., diverges.

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$\sum_{n=2}^{\infty} \frac{1}{n^2+4}$ within .1 of exact.

$-\int_{N+1}^{\infty} \geq -S'_N \geq -\int_{N}^{\infty}$

$f(n) = \frac{1}{n^2+4}$ is decreasing $\forall\, n \geq 1$

$S_N + \sum_{n=N+1}^{\infty} = S'$

$0.1 > \boxed{S' - \int_{N+1}^{\infty}} \geq \boxed{S' - S_N} \geq S' - \int_{N}^{\infty}$

$\| \sum_{k=N+1}^{\infty} a_k$

$\int_{n}^{\infty} \frac{1}{x^2+4}\, dx = \lim_{b\to\infty} \frac{1}{2}\arctan\left(\frac{x}{2}\right)\Big]_{n}^{b}$

$$= \lim_{b \to \infty} \left[\frac{1}{2} \arctan\left(\frac{b}{2}\right) - \frac{1}{2} \arctan\left(\frac{n}{2}\right) \right]$$

$$= \frac{1}{2} \cdot \frac{\pi}{2} - \frac{1}{2} \arctan\left(\frac{n}{2}\right) \quad \underset{<}{\text{want}} \quad 0.1$$

$$\Longrightarrow -\frac{1}{2} \arctan\left(\frac{n}{2}\right) < 0.1 - \frac{\pi}{4}$$

$$\arctan\left(\frac{n}{2}\right) > 2\left[\frac{\pi}{4} - .1\right]$$

$$\frac{n}{2} > \tan\left(2\left[\frac{\pi}{4} - .1\right]\right)$$

$$n > 2\tan\left(2\left[\frac{\pi}{4} - .1\right]\right) \approx$$

$$2 \cdot \tan\left(2 \cdot \left(\frac{\mathrm{Pi}}{4} - 0.1\right)\right)$$

9.866309752

evalf(%)

9.866309752

Pick $n = 10$ for S'_n

$\lim_{n \to \infty} \arctan\left(\frac{n}{2}\right) = \frac{\pi}{4}$, not $\frac{\pi}{8}$.

Input & output of $\arctan\left(\frac{n}{2}\right)$ was mixed up by teacher.

Back to §10.6

Alternating Series Test ✓

$\boxed{\text{TIS}}$ $\sum_{k=1}^{n} (-1)^{k+1} a_k$ is within a_{n+1} of S

i.e. $|S - S_n| \leq a_{n+1}$

$$evalf\left(\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} - \sum_{k=1}^{5} \frac{(-1)^{k+1}}{k}\right)$$

-0.0901861527

$$evalf\left(\frac{1}{6}\right)$$

0.1666666667

$a_6 \geq |S - S_5|$

$$evalf\left(\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} - \sum_{k=1}^{6} \frac{(-1)^{k+1}}{k}\right)$$

0.0764805139

$$evalf\left(\frac{1}{7}\right)$$

0.1428571429

10 $\qquad (-1)^{n+1}\, 2n$

$10-8$

$10-8+7$

$10\,[-8+7-6]$

$10\,[-8+7-6+5]$

$10\,[-8+7-6+5-4]$

$10-8+[7-6+5-4+3]$

Now

Corollary (ies)

To get error less than M, Make $a_n < M$. Sum to $n-1$.

$$\begin{cases} S_n < L < S'_{n+1} & \text{If } (-1)^{n+1} a_n < 0 \\ \text{OR} & \\ S'_{n+1} < L < S'_n & \text{If } (-1)^{n+1} a_n > 0 \end{cases}$$

$$\sum_{k=1}^{20} (-1)^{k+1} \frac{1}{k} < L < \sum_{k=1}^{21} (-1)^{k+1} \frac{1}{k}$$

$$\left| L - \sum_{k=1}^{21} (-1)^{k+1} \frac{1}{k} \right| < \frac{1}{22}$$