$$\frac{a_{n+1}}{a_n} = a_{n+1} \cdot \frac{1}{a_n}$$

$$5! = 5 \cdot 4!$$
$$(n+1)! = (n+1)\,n!$$

38 $$\frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} = a_{n+1} \cdot \frac{1}{a_n}$$

$$= \frac{(n+1)}{(n+1)^{n+1}} \cdot n^n = \frac{\cancel{n+1}}{\cancel{(n+1)}(n+1)^n} \cdot n^n = \frac{n^n}{(n+1)^n}$$

$$(n+1)^n = n^n\left(1+\frac{1}{n}\right)^n$$

$$\underbrace{(n+1)(n+1)\cdots(n+1)}_{n \text{ factors}} = \underbrace{n\left(1+\frac{1}{n}\right) n\left(1+\frac{1}{n}\right) \cdots n\left(1+\frac{1}{n}\right)}_{n \text{ factors}}$$

$$= n^n\left(1+\frac{1}{n}\right)^n$$

$$= \frac{n^n}{(n+1)^n} = \frac{n^n}{n^n\left(1+\frac{1}{n}\right)^n} = \frac{1}{\left(1+\frac{1}{n}\right)^n} \xrightarrow{n \to \infty} \frac{1}{e} < 1$$

$$e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \lim_{x\to 0}(1+x)^{\frac{1}{x}}$$

$$\ln x = \int \frac{dx}{x} \quad \text{for } x > 0$$

39 $\sum_{2}^{\infty} \frac{n}{(\ln(n))^n}$

Root Test : $\sqrt[n]{\frac{n}{(\ln(n))^n}} = \frac{\sqrt[n]{n}}{\sqrt[n]{(\ln(n))^n}} = \frac{\sqrt[n]{n}}{\ln(n)} \xrightarrow{n\to\infty} 0$

$\sqrt[n]{n} \to 1$

$\ln(n) \to \infty$

$y = \lim_{n\to\infty} \sqrt[n]{n} = \lim_{n\to\infty} n^{\frac{1}{n}}$

wolframalpha

$$\sum_{1}^{\infty} \frac{n^n}{(2^n)^2} = \sum_{1}^{\infty} \frac{n^n}{2^{2n}} = \sum_{1}^{\infty} \frac{n^n}{(2^2)^n}$$

$$\sqrt[n]{\frac{n^n}{(2^2)^n}} = \frac{n}{2^2} \xrightarrow{n \to \infty} \infty$$

Diverges

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

$$\frac{1}{n+1} \cdot \frac{n}{1} = \frac{n}{n+1} \xrightarrow{n \to \infty} 1 \quad \text{Inconclusive.}$$

Mar 5-12:30 PM

$$\sum \frac{(3n)!}{n!\,(n+1)!\,(n+2)!}$$

Ratio Test

$$\frac{(3(n+1))!}{\cancel{(n+1)!}\,\cancel{(n+2)!}\,(n+3)!} \cdot \frac{n!\,\cancel{(n+1)!}\,\cancel{(n+2)!}}{(3n)!}$$

$$= \frac{(3n+3)!\,\cancel{n!}}{(n+3)(n+2)(n+1)\cancel{n!}\,(3n)!}$$

$$= \frac{(3n+3)(3n+2)(3n+1)\cancel{(3n)!}}{(n+3)(n+2)(n+1)\,\cancel{(3n)!}}$$

$$= \frac{3^3 n^3 \left(1+\frac{1}{n}\right)\left(1+\frac{2}{3n}\right)\left(1+\frac{1}{3n}\right)}{n^3\left(1+\frac{3}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{1}{n}\right)}$$

$$3n+3 = 3n\left(1+\frac{1}{n}\right)$$

$$3n+2 = 3n\left(\frac{3n}{3n}+\frac{2}{3n}\right)$$

$$= \frac{3^3\left(1+\frac{1}{n}\right)\left(1+\frac{2}{3n}\right)\left(1+\frac{1}{3n}\right)}{\left(1+\frac{3}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{1}{n}\right)} \xrightarrow{n\to\infty} 27$$

Diverges.

§10.6

T14 $a_n \geq 0$

$a_n \geq a_{n+1} \geq \dots \ \forall n \geq N$ (eventually)

$a_n \longrightarrow 0$

Then $\sum (-1)^{n+1} a_n$ converges

$\boxed{E}$ $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{.002}}$

$n^{.002} = n^{\frac{2}{1000}} = n^{\frac{1}{500}} = \sqrt[500]{n}$

increases without bound.

Proof: Let $M > 0$

Scratch:

Want $\sqrt[500]{n} > M$

$n > M^{500}$ does the trick.

$n^{.002} \xrightarrow{n \to \infty} \infty \implies$

$\boxed{\frac{1}{n^{.002}} \xrightarrow{n \to \infty} 0}$ ✓

Mar 5-12:42 PM

Need $a_n \geq a_{n+1} \geq \dots$ eventually.

$$f(x) = x^{-.002}$$

$$f'(x) = -.002\, x^{-1.002} = -\frac{.002}{x^{1.002}} < 0 \quad \forall x > 0$$

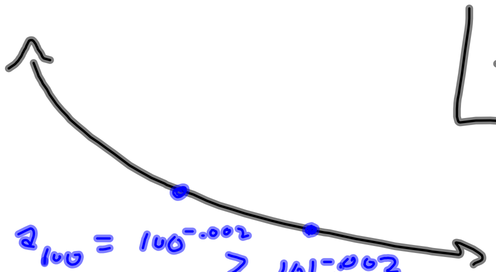
$$f(n) = n^{-.002}$$

$$f'(n) = -.002\, n^{-1.002}$$

$$\boxed{a_n \geq a_{n+1} \geq \dots \quad \forall\, n \geq 1} \checkmark$$

$$\boxed{a_n = \frac{1}{n^{.002}} > 0 \quad \forall\, n} \checkmark$$

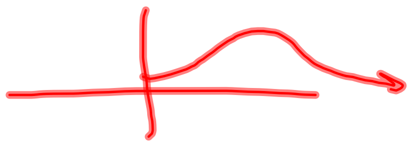
Series Converges.



$\sum (-1)^n \frac{1}{n^2+1} = \sum (-1)^n (n^2+1)^{-1}$

$f'(x) = -1(n^2+1)^{-2}(2n$

Not quite

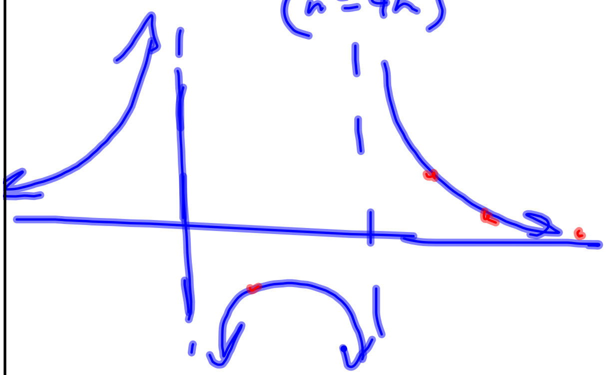


$\sum (-1)^n \frac{1}{n^2-4n} = \sum (-1)^n (n^2-4n)^{-1}$

$f'(n) = -(n^2-4n)^{-2}(2n-4) \overset{SET}{=} 0 \Rightarrow n=2$

$-\frac{2n-4}{(n^2-4n)^2} = -\frac{2(n-2)}{(n(n-4))^2} < 0$ for $n > 2$

Not defined @ $n=4$



1, 5, 9, 13, ..., 53, 55, 57

Clint : 1, 57

Adam 5, 55

Ryan 9, 53

Zach 13, 49

Jon 17, 45

Derek 21, 41

Josh 25, 37

Karla 29, 33

10.5

3, 7, 12, 14, 15,

17, 21, 27, 33,

45, 57, 61

Mar 5-1:00 PM