(38) $\sum \frac{n!}{n^n}$ (40) $\sum \frac{n}{(\ln(n))^{n/2}}$

$$\frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!}$$

$$= \frac{(n+1)\, n^n}{(n+1)^{n+1}} = \frac{(n+1)\, n^n}{n^{n+1}\left(1+\frac{1}{n}\right)^{n+1}} = \frac{(n+1)}{n\left(1+\frac{1}{n}\right)^{n+1}}$$

$$= \frac{n+1}{n\left(1+\frac{1}{n}\right)^n\left(1+\frac{1}{n}\right)} = \frac{n\left(1+\frac{1}{n}\right)}{n\left(1+\frac{1}{n}\right)^n\left(1+\frac{1}{n}\right)} = \frac{1}{\left(1+\frac{1}{n}\right)^n}$$

$$\xrightarrow{n \to \infty} \frac{1}{e} < 1 \text{ converges!}$$

Mar 4-12:11 PM

$$(n+1)! = (n+1)\underbrace{(n)(n-1)(n-2)\cdots(3)(2)(1)}_{n!}$$

$$=(n+1)\cdot n!$$

$$\frac{(n+1)!}{(n+1)^{n+1}}\cdot\frac{n^n}{n!} = \frac{(n+1)\,n^n}{(n+1)^{n+1}}$$

$$= \frac{(n+1)\,n^n}{(n+1)^n\,(n+1)} = \frac{n^n}{(n+1)^n} = \frac{n^n}{n^n\left(1+\frac{1}{n}\right)^n} = \frac{1}{\left(1+\frac{1}{n}\right)^n}$$

$$\left(\underbrace{(n+1)(n+1)\cdots(n+1)}_{n \text{ of 'em}} = n^n\left(1+\tfrac{1}{n}\right)\cdots\left(1+\tfrac{1}{n}\right) \qquad n+1 = n\left(1+\tfrac{1}{n}\right)\right)$$

$$= \frac{1}{\left(1+\frac{1}{n}\right)^n} \xrightarrow{n\to\infty} \frac{1}{e}$$

$$\underline{y = \left(1+\tfrac{1}{n}\right)^n} \qquad \ln y = \ln\left(\left(1+\tfrac{1}{n}\right)^n\right) \Rightarrow$$

$$\underline{\ln y = n\ln\left(1+\tfrac{1}{n}\right)} \xrightarrow{n\to\infty} \infty\cdot 0$$

L'Hopital's:

$$\frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}} \xrightarrow{n\to\infty} \frac{0}{0}$$

$$\lim_{n\to\infty} \frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}} \overset{l'H}{=} \lim_{n\to\infty} \frac{\frac{-\frac{1}{n^2}}{1+\frac{1}{n}}}{-\frac{1}{n^2}} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = 1$$

$$\ln y = 1$$

$$y = e^1$$

$$\boxed{\begin{array}{l} (1+x)^{\frac{1}{x}} \xrightarrow{x\to 0} e \\ \left(1+\frac{1}{x}\right)^x \xrightarrow{x\to\infty} e \end{array}}$$

Future value of \$P invested at r rate of interest, compounded m times per year for t years.

$$A = P\left(1+\frac{r}{m}\right)^{mt} \xrightarrow{m\to\infty} Pe^{rt}$$

$$mt = \frac{m}{r}\cdot rt$$

$$\frac{r}{m} = \frac{1}{\frac{m}{r}}$$

$$P\left(1+\frac{1}{m/r}\right)^{(m/r)rt} \xrightarrow{m/r\to\infty} Pe^{rt}$$

$$\sum \frac{n}{(\ln n)^{n/2}}$$

$$\sqrt[n]{\frac{n}{\ln(n)^{n/2}}} = \frac{n^{\frac{1}{n}}}{\ln(n)^{n/2n}} = \frac{n^{\frac{1}{n}}}{\ln(n)^{\frac{1}{2}}} = 0$$

$$\lim_{n\to\infty} n^{\frac{1}{n}} = 1 \qquad y = \lim_{n\to\infty} n^{\frac{1}{n}}$$

$$\lim_{n\to\infty} \sqrt{\ln(n)} = \infty \qquad \ln y = \ln\left(\lim_{n\to\infty} n^{\frac{1}{n}}\right)$$

$$= \lim_{n\to\infty}\left(\ln\left(n^{\frac{1}{n}}\right)\right)$$

$$= \lim_{n\to\infty} \frac{\ln n}{n} \overset{L'H}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$$

$$e^{\ln y} = e^0$$

$$y = 1$$

Mar 4-12:31 PM

$$\int_0^1 \frac{4r\,dr}{\sqrt{1-r^4}} = 2\cdot\int_0^1 \frac{\overset{2r\,dr}{4r\,dr}}{\sqrt{1-(r^2)^2}} = 2\int_0^1 \frac{du}{\sqrt{1-u^2}}$$

$u = r^2$ $\quad$ $du = 2r\,dr$

$u(0) = 0^2$

$u(1) = 1^2$

$u = \sin\theta$ $\quad$ $du = \cos\theta\,d\theta$

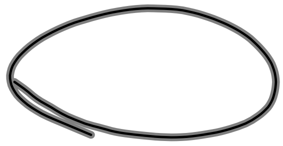
$\theta = \arcsin u$ $\quad$ $\arcsin(0) = 0$

$\arcsin(1) = \frac{\pi}{2}$

This gives

$$2\int_0^{\frac{\pi}{2}} \frac{\cos\theta\,d\theta}{\sqrt{1-\sin^2\theta}} = 2\int_0^{\frac{\pi}{2}} d\theta = \pi$$

$$\int_{-\infty}^{\infty} \frac{x\,dx}{(x^2+4)^{\frac{3}{2}}} = 0$$



Symmetric Interval, odd function.

$$\int_{-2}^{2} (x^2+5x-7)\,dx \neq 0$$

$$f(x) = \frac{x}{(x^2+4)^{3/2}}$$

$$f(-x) = \frac{(-x)}{((-x)^2+4)^{3/2}} = -\frac{x}{(x^2+4)^{3/2}} = -f(x)$$

even · even = even

odd · even = odd

$\frac{\text{even}}{\text{odd}}$ = odd, etc.

Read §10.6

$\sum (-1)^n a_n$ converges if $a_n \longrightarrow 0$

<u>Rearrange</u> to any sum you want!

$\rightarrow$ Rearrangements are deadly.