10.2 Repeating Decimals

$1.2379$

$= 1\times 10^{0} + 2\times 10^{-1} + 3\times 10^{-2} + 7\times 10^{-3} + 9\times 10^{-4}$

$$\boxed{\begin{array}{rrl} & 234.\overline{234} & = 1000x \\ - & 0.\overline{234} & = x \\ \hline & 234 & = 999x \\ & \frac{234}{999} & = x \end{array}}$$

$.\overline{234} = .234234234\ldots$

$= .234 + .000234 + .000000234 + \ldots$

$= 234\times 10^{-3} + 234\times 10^{-6} + 234\times 10^{-9} + \ldots$

$= (234)(10^{-3}) + (234)(10^{-3})^2 + (234)(10^{-3})^3 + \ldots$

$=$

$(234)(10^{-3})(10^{-3})$

$(234)(10^{-3})(10^{-3})^2$

$= \sum_{n=1}^{\infty} .234\,(10^{-3})^{n-1}$

$S' = \frac{a}{1-r}$

$a = .234$

$r = 10^{-3} = \frac{1}{1000}$

$= \dfrac{.234}{1-\frac{1}{1000}} = \dfrac{.234}{\frac{999}{1000}}$

$= (.234)\left(\frac{1000}{999}\right) = \frac{234}{999}$

Our Goal: To represent an infinitely differentiable function as a power series.

$S = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \cdots$

$S_n = b_0 + b_1 x + b_2 x^2 + \cdots + b_n x^n$ is a polynomial of degree $n$

It turns out we can do this, i.e.

$$e^x = b_0 + b_1 x + b_2 x^2 + \cdots + b_n x^n + \cdots$$
$$= \sum_{n=0}^{\infty} b_n x^n = f(x)$$

If these are the same, then

$f(0) = e^0 = 1 = b_0 \qquad f(x) \approx 1$

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$f'(x) = e^x = 0 + b_1 + 2b_2 x + 3b_3 x^2 + \dots$

$f'(0) = e^0 = b_1 = 1 \qquad f(x) = 1 + 1x$

$f''(x) = 2b_2 + 3 \cdot 2 b_3 x + 4 \cdot 3 b_4 x^2 + 5 \cdot 4 b_5 x^3 + \dots$

$f''(0) = e^0 = 1 = 2b_2 \implies \frac{1}{2} = b_2$

$f(x) \approx 1 + 1x + \frac{1}{2}x^2$

$f'''(x) = e^x = 3 \cdot 2 b_3 + 4 \cdot 3 \cdot 2 b_4 x + 5 \cdot 4 \cdot 3 b_5 x^2 + \dots$

$f'''(0) = e^0 = 3 \cdot 2 \cdot b_3 = 1 \implies b_3 = \frac{1}{3 \cdot 2}$

$f(x) \approx 1 + x + \frac{1}{2}x^2 + \frac{1}{3 \cdot 2}x^3$

is $3^{rd}$-degree Taylor Polynomial for $f(x) = e^x$

$\vdots$

$f^{(4)}(x) = 4 \cdot 3 \cdot 2 b_4 + 5 \cdot 4 \cdot 3 \cdot 2 b_5 x + \dots$

$f^{(4)}(0) = 4 \cdot 3 \cdot 2 b_4 = e^0 = 1$

$b_4 = \frac{1}{4 \cdot 3 \cdot 2} = \frac{1}{4!}$

$b_5 = \frac{1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{1}{5!}$

$b_6 = \frac{1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{1}{6!}$

So, we believe

$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \dots$

Ratio & Root Tests are the biggies

for Power Series, especially Ratio Test.

FOR series with positive terms.

Ratio: want $\frac{a_{n+1}}{a_n} \xrightarrow{n \to \infty} c < 1.$

Root: want $\sqrt[n]{a_n} \xrightarrow{n \to \infty} c < 1$

$c = 1$ is inconclusive

$c > 1$ diverges.

55 $\sum \frac{2^n n! n!}{(2n)!}$

$$\frac{\frac{2^{n+1}(n+1)!(n+1)!}{(2(n+1))!}}{\frac{2^n n! n!}{(2n)!}} =$$

$$\frac{5\cdot \cancel{4\cdot 3\cdot 2}}{\cancel{4\cdot 3\cdot 2}}$$

$$\frac{2^{n+1}(n+1)!(n+1)!}{(2(n+1))!}\cdot\frac{(2n)!}{2^n n! n!} = \frac{2(n+1)(n+1)}{(2n+2)(2n+1)}$$

$$\frac{(2n)!}{(2n+2)!} = \frac{\cancel{(2n)!}}{(2n+2)(2n+1)\cancel{(2n)!}}$$

$$= \frac{2\cancel{n^2}(1+\frac{1}{n})(1+\frac{1}{n})}{\cancel{n^2}(2+\frac{2}{n})(2+\frac{1}{n})} \xrightarrow{n\to\infty} \frac{2(1)(1)}{(2)(2)} = \frac{1}{2} < 1$$

Converges!

§10.5 #s 3, 7, 12!, 14!, 15, 17, 21, 27, 33, 45, 57, 61