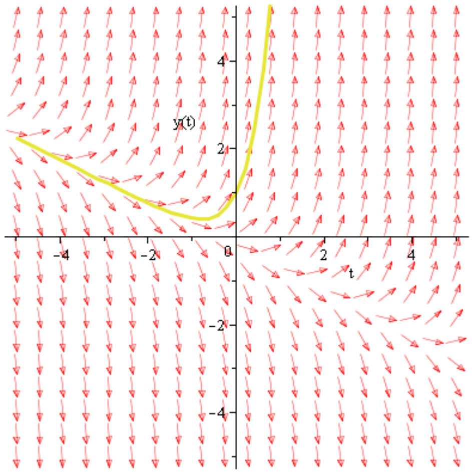
$diff(y(t), t)$ is the derivative of y with respect to t. Use the ^ key to do exponents. So the $y(t)^2$ was obtained by typing y(t)^2. This is just one command, which you should be able to hack into any of the problems you will see in homework.

$y' = x + 2y \implies y' - 2y = x \qquad e^{\int -2\,dx} = v(x)$

$with(DEtools)$:

$DEplot(diff(y(t), t) = t + 2 \cdot y(t), y(t), t = -5..5, y(t) = -5..5, arrows = medium, dirgrid = [20, 20], [[y(0) = 1]])$



$\Delta x = 1$

Conjecture: $\lim_{n\to\infty} \frac{a_n+6}{a_n+2} = 2$

$a_1 = 1$, $a_{n+1} = \frac{a_n+6}{a_n+2}$

Confirm: Note: all a_n's > 0 eventually.

Assume the limit does exist

$\lim_{n\to\infty} \frac{a_n+6}{a_n+2} = L \longrightarrow$

$\frac{L+6}{L+2} = L$

$L+6 = L^2+2L$

$L^2+L-6=0$

$(L+3)(L-2)=0$

$\cancel{L=-3}$ or $\boxed{L=2}$

$L>0$

Feb 25-12:14 PM

Bounded Sequence

Least Upper Bound

Greatest Lower Bound

Monotone sequence

Nondecreasing $a_1 \leq a_2 \leq a_3 \leq \ldots$

Nonincreasing $a_1 \geq a_2 \geq a_3 \geq \ldots$

Your 1st Monotone Convergence Theorem

A nondecreasing sequence, bdd above, converges.

A nonincreasing sequence, bdd below, converges.

§10.1 #s 6, 10, 16, 18, 30, 36, 46, 92.

10.1 #36

$a_n = (-1)^n \left(1 - \frac{1}{n}\right) = (-1)^n \left(\frac{n-1}{n}\right)$

$a_1 = 0$

$a_2 = \frac{1}{2}$

$a_3 = -\frac{2}{3}$

$a_4 = \frac{3}{4}$

$a_5 = -\frac{4}{5}$

$a_6 = \frac{5}{6}$

$a_7 = -\frac{6}{7}$

$\vdots$

Bdd above & below by ± 1

Try $\varepsilon = \frac{1}{2}$ to Prove it Diverges.

Diverges.

$a_{2n} \longrightarrow 1$

$a_{2n+1} \longrightarrow -1$

Since $a_{2n} \longrightarrow 1$

Try $L = 1$

But then all the a_{2n+1}'s are more than $\varepsilon = \frac{1}{2}$ away from your L.

$\{a_n\}$ 10.1

$\sum_{n=1}^{\infty} a_n$ 10.2 $\sum_{k=1}^{\infty} a_k$

1st foray into infinite series:

$\sum_{k=1}^{\infty} ar^{k-1}$ Geometric Series.

$S_n = n^{\text{th}}$ partial sum $= \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} ar^{k-1}$

To study series, we study the sequence of n^{th} partial sums: $\{S_n\}$

$S_1 = a$

$S_2 = a + ar$

$S_3 = a + ar + ar^2$

$\vdots$

Telescopes!

$$\begin{aligned} S_n &= a + \cancel{ar} + \cancel{ar^2} + \cdots + \cancel{ar^{n-1}} \\ -\; rS_n &= -(\cancel{ar} + \cancel{ar^2} + \cdots + \cancel{ar^{n-1}} + ar^n) \end{aligned}$$

$S_n - rS_n = a - ar^n$

$S_n(1-r) = a(1-r^n)$

$S_n = \dfrac{a(1-r^n)}{1-r}$ Nice, tidy, closed form expression.

$$\therefore \sum_{k=1}^{\infty} a r^{k-1} = \boxed{\lim_{n\to\infty} \frac{a(1-r^n)}{1-r}} = \lim_{n\to\infty} S_n$$

converges to $\frac{a(1-0)}{1-r}$ or $\frac{a}{1-r}$ only if $|r|<1$

$$\sum_{k=1}^{\infty} 2\cdot 3^{k-1} \quad \text{Diverges} \qquad r=3>1$$

$$\sum_{k=1}^{\infty} 2\cdot\left(\tfrac{1}{3}\right)^{k-1} \qquad a=2 \quad r=\tfrac{1}{3} \qquad \frac{2}{1-\frac{1}{3}} = \frac{2}{\frac{2}{3}} = 2\cdot\frac{3}{2} = \boxed{3}$$

$$\sum_{k=5}^{\infty} 2\cdot\left(\tfrac{1}{3}\right)^{k-1} = 2\cdot\left(\tfrac{1}{3}\right)^4 + 2\cdot\left(\tfrac{1}{3}\right)^5 + 2\cdot\left(\tfrac{1}{3}\right)^6 + \cdots$$

$$= 2 + 2\cdot\tfrac{1}{3} + 2\cdot\left(\tfrac{1}{3}\right)^2 + 2\cdot\left(\tfrac{1}{3}\right)^3 + \sum_{k=5}^{\infty} 2\cdot\left(\tfrac{1}{3}\right)^{k-1}$$

$$-\left(2 + 2\cdot\tfrac{1}{3} + 2\cdot\left(\tfrac{1}{3}\right)^2 + 2\cdot\left(\tfrac{1}{3}\right)^3\right)$$

$$= \sum_{k=1}^{\infty} 2\cdot\left(\tfrac{1}{3}\right)^{k-1} - \left(2 + 2\cdot\tfrac{1}{3} + 2\cdot\left(\tfrac{1}{3}\right)^2 + 2\cdot\left(\tfrac{1}{3}\right)^3\right)$$

$$2\cdot\left(\tfrac{1}{3}\right)^4 + 2\cdot\left(\tfrac{1}{3}\right)^5 + 2\cdot\left(\tfrac{1}{3}\right)^6 + \cdots$$

$a \qquad ar \qquad ar^2 \qquad 2\left(\tfrac{1}{3}\right)^5 = 2\left(\tfrac{1}{3}\right)^4 \cdot \tfrac{1}{3}$

$a = 2\left(\tfrac{1}{3}\right)^4$

$r = \tfrac{1}{3}$

$$S = \boxed{\frac{2\left(\frac{1}{3}\right)^4}{1-\frac{1}{3}}} = \frac{2\cdot\frac{1}{81}}{\frac{2}{3}} = \frac{1}{81}\cdot 3 = \frac{1}{27}$$

$$= \sum_{k=1}^{\infty} 2\cdot\left(\tfrac{1}{3}\right)^{k-1} - \left(2 + 2\cdot\tfrac{1}{3} + 2\cdot\left(\tfrac{1}{3}\right)^2 + 2\cdot\left(\tfrac{1}{3}\right)^3\right)$$

$$= \frac{2}{1-\frac{1}{3}} - 2\left(1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27}\right)$$

$$= 3 - 2\left(\frac{27+9+3+1}{27}\right) = 3 - 2\left(\frac{40}{27}\right) =$$

$$= 3 - \frac{80}{27} = \frac{81-80}{27} = \frac{1}{27}$$ Sweet!

If $\sum a_n$ converges then $a_n \longrightarrow 0$

Test for Divergence

If $a_n \not\longrightarrow 0$ then $\sum a_n$ diverges

$$\left.\begin{aligned} \sum(a_n \pm b_n) &= \sum a_n \pm \sum b_n \\ \sum k a_n &= k \sum a_n \end{aligned}\right\} \text{ If } \sum a_n \text{ \& } \sum b_n \text{ converge.}$$

Note: Sometimes $\sum(a_n + b_n)$ converges when Neither $\sum a_n$ nor $\sum b_n$ converge.

§10.2 #s 6, 12, 14, 18, 24, 28, 30, 36, 40?, 44