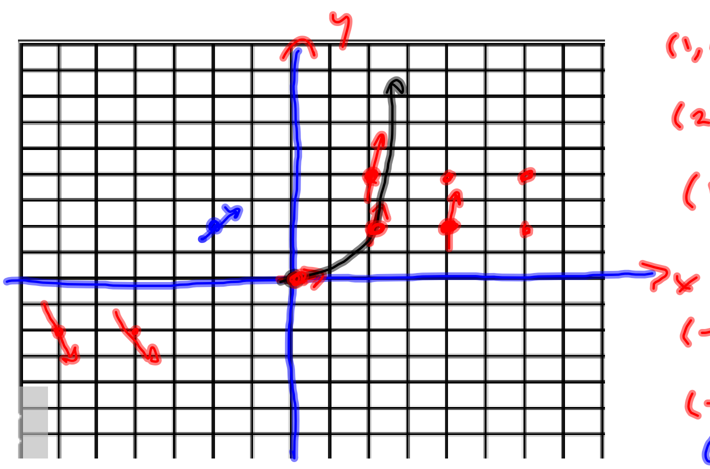
$$\frac{dy}{dx} = x + 2y$$

$$y' = x + 2y$$

$y' - 2y = x$ is linear 1^{st} order equation in standard form.



(1,1) $y' = 1 + 2(1) = 3$

(2,1) $y' = 2 + 2(1) = 4$

(1,2) $y' = 1 + 2(2) = 5$

(-1,1) $y' = -1 + 2(1) = 1$

(-3,-1) $y' = -3 + 2(-1) = -5$

(-2,-1) $y' = -2 + 2(-1) = -4$

(0,0) $y' = 0$

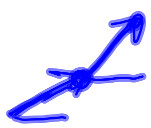
$\Delta x = 1$

Use tangent line to proceed to next point

Plug it into DE. Get new y'

Make little tangent line,

use it to move to next point



Goal: Get the LHS to be something we can integrate. Make it look like the result of a product rule.

$$f'y + fy' = \text{stuff}$$

$$(fy)' = \text{stuff}$$

$$\int (fy)' = \int \text{stuff}$$

$$fy = \int \text{stuff}$$

$$y = \frac{1}{f}\int \text{stuff}$$

Call $v(x)$ our "integrating factor"

$$v(x)y' + P(x)v(x)y = v(x)Q(x)$$

(Started with $y' + P(x)y = Q(x)$)

want $v(x)y' + P(x)v(x)y$ to be

$$\frac{d}{dx}(v(x)y)$$

$$= v'(x)y + v(x)y' = v(x)y' + P(x)v(x)y$$

$$v'(x)y = P(x)v(x)y$$

$$v'(x) = P(x)v(x)$$

$$\frac{\frac{dv}{dx}}{v(x)} = \frac{v'(x)}{v(x)} = P(x)$$

$$\int \frac{dv}{v(x)} = \int P(x)\,dx$$

$$\ln v(x) = \int P(x)\,dx$$

$v(x) = e^{\int P(x)dx}$ is the integrating factor.

$$y' - 2y = x \qquad y' + P(x)y = Q(x)$$

$$P(x) = -2$$

$$\boxed{e^{\int P(x)\,dx}} = e^{\int -2\,dx} = \boxed{e^{-2x+C}}$$

$$= e^{-2x}e^{C} = \boxed{Ke^{-2x}} \text{ for some } K$$

our integrating factor.

$v(x)$ made

$$y' + P(x)y = Q(x)$$

into

$$\underbrace{v(x)y' + P(x)v(x)y}_{} = v(x)Q(x)$$

$$\int (v(x)y)' = \int v(x)Q(x)$$

$$v(x)y = \int v(x)Q(x)\,dx$$

$$y = \frac{1}{v(x)}\int v(x)Q(x)\,dx \text{ is the solution!}$$

$$y = \frac{1}{Ke^{-2x}}\int Ke^{-2x}\,x\,dx$$

$u = x \quad du = dx$

x	$+$	e^{-2x}
1	$-$	$-\frac{1}{2}e^{-2x}$
0		$\frac{1}{4}e^{-2x}$

$$= e^{2x}\left[-\tfrac{1}{2}xe^{-2x} - \tfrac{1}{4}e^{-2x} + C\right]$$

$$= -\tfrac{1}{2}x - \tfrac{1}{4} + Ce^{2x} = y$$

$$y' = -\tfrac{1}{2} + 2Ce^{2x}$$

$$y' - 2y = -\tfrac{1}{2} + 2Ce^{2x} - 2\left[-\tfrac{1}{2}x - \tfrac{1}{4} + Ce^{2x}\right]$$

$$= -\tfrac{1}{2} + 2Ce^{2x} + x + \tfrac{1}{2} - 2Ce^{2x}$$

$$= x$$

$$y' + P(x)\,y = Q(x)$$

$$y = \frac{1}{v(x)} \int v(x)\,Q(x)\,dx,$$

where $v(x) = e^{\int P(x)\,dx}$

① $(1+x)\,y' + y = \sqrt{x}$

$$y' + \frac{1}{x+1}\,y = \frac{\sqrt{x}}{x+1}$$

② $x\,y' = \frac{\cos x}{x} - 2y \qquad x > 0$

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$$\int e^{-\theta} \sin\theta \, d\theta$$

$$\begin{pmatrix} u = e^{-\theta} & du = -e^{-\theta} d\theta \\ dv = \sin\theta \, d\theta & v = -\cos\theta \end{pmatrix}$$

$$= uv - \int v \, du = -e^{-\theta}\cos\theta - \int (-e^{-\theta})(-\cos\theta) \, d\theta$$

$$= -e^{-\theta}\cos\theta - \int e^{-\theta} \cos\theta \, d\theta$$

$$\begin{pmatrix} u = e^{-\theta} & du = -e^{-\theta} d\theta \\ dv = \cos\theta \, d\theta & v = \sin\theta \end{pmatrix}$$

$$= -e^{-\theta}\cos\theta - \left[uv - \int v \, du \right]$$

$$= -e^{-\theta}\cos\theta - \left[e^{-\theta}\sin\theta + \int e^{-\theta} \sin\theta \, d\theta \right]$$

$$\therefore \int e^{-\theta}\sin\theta \, d\theta = \frac{-e^{-\theta}\cos\theta - e^{-\theta}\sin\theta}{2} + C$$

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