Find $p \ni \ldots$

$\boxed{\ldots \int_1^2 \frac{dx}{x(\ln x)^p} \text{ converges}}$ Homework

$\ldots \int_2^\infty \frac{dx}{x(\ln x)^p}$ converges DONE: $p > 1$ Does it.

$\int \frac{dx}{x(\ln x)^p} = ?$

Let $u = \ln x$:
$du = \frac{1}{x}dx$ This gives

$\int x^n dx =$

$\int (\ln x)^{-p} \cdot \frac{1}{x} dx = \int u^{-p} du$

$= \frac{1}{1-p} u^{1-p} + C$, provided $p \neq 1$.
Need to look @ $p = 1$, separately.

Then $\int_2^\infty \frac{dx}{x(\ln x)^p} = \lim_{t\to\infty} \int_2^t \frac{dx}{x(\ln x)^p} = \lim_{t\to\infty} \int_{\ln 2}^{\ln t} u^{-p} du$

$u = \ln x$
$u(2) = \ln 2$
$u(t) = \ln t$

$\lim_{t\to\infty} \left(\frac{1}{1-p} u^{1-p}\right)_{\ln 2}^{\ln t} =$

$$= \frac{1}{1-p} \lim_{t\to\infty} \left[(\ln t)^{1-p} - (\ln 2)^{1-p} \right]$$

$$= \frac{1}{1-p} \lim_{t\to\infty} (\ln t)^{1-p} - \frac{1}{1-p} (\ln 2)^{1-p}$$ ✓ Real

Break into sub-cases

① $p \neq 1$

ⓐ $p > 1 \longrightarrow$

ⓑ $p < 1 \; \cancel{\longrightarrow}$

② $p = 1 \; \cancel{\longrightarrow}$

ⓐ $p > 1$, then $1-p < 0$

and $\lim_{t\to\infty} (\ln t)^{1-p}$

$= \lim_{t\to\infty} \frac{1}{(\ln t)^b}$ where $b > 0$

$= 0$ if $b > 0$ (i.e., $p > 1$)

ⓑ $p < 1$:

$-p > -1$

$-p+1 > 0$

$1-p > 0$

$\lim_{t\to\infty} (\ln t)^{1-p} = \lim_{t\to\infty} (\ln t)^b$ where $b > 0$

$= \infty$

② $\displaystyle\int \frac{dx}{x \ln x} = \int \frac{du}{u}$, where $u = \ln x$

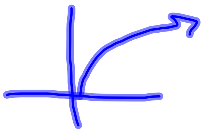
which gives $\ln|\ln x| + C$

$$\int_2^\infty \frac{dx}{x\ln x} = \lim_{t\to\infty} \int_{\ln 2}^{\ln t} \frac{du}{u} = \lim_{t\to\infty} \ln|u| \Big]_{\ln 2}^{\ln t}$$

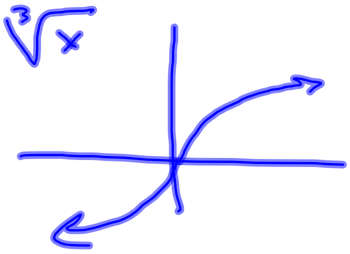
$$= \lim_{t\to\infty} \left[\ln|\ln t| - \ln(\ln 2) \right] = \infty$$

$b > 1$ Done.

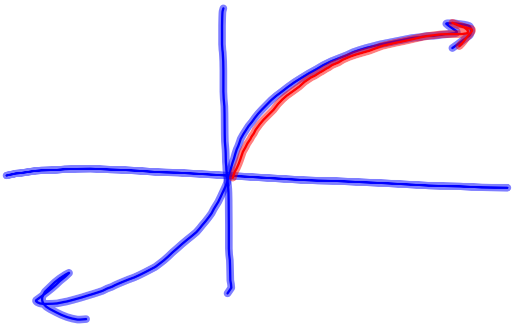
$\sqrt{x}$



$\sqrt[3]{x}$



$\sqrt[523]{x}$



$$\sqrt[n]{x} \xrightarrow{x \to \infty} \infty$$

Increases without Bound means eventually $f(x) > M$ for any $M > 0$ you might name.

"Eventually" means there is an N such that for any $x > N$, the property holds

$\frac{1}{x}$ is eventually close to zero.

Let $\varepsilon > 0$

want $\left|\frac{1}{x} - 0\right| < \varepsilon$ eventually.

$$\frac{1}{x} < \varepsilon \Rightarrow 1 < \varepsilon x \Rightarrow \frac{1}{\varepsilon} < x$$

Let $N = \frac{1}{\varepsilon}$. Then $x > N$, we have

$$\left|\frac{1}{x} - 0\right| = \boxed{\frac{1}{x} <} \frac{1}{N} = \frac{1}{\frac{1}{\varepsilon}} = \boxed{\varepsilon}$$

$\sqrt[n]{x}$ grows without bound.

Let $M > 0$ be given.

want $\sqrt[n]{x} > M$

$$x > M^n \equiv N$$

Let $x > N$. Then $\sqrt[n]{x} > \sqrt[n]{N} = M$

Really want to internalize

$$\ln(\ln x) \xrightarrow{x \to \infty} \infty$$

$$\int_0^3 \frac{x^5}{\sqrt{x^2-1}}\,dx$$

x = 1 is a prob ✓

$x = \sec\theta$
$dx = \sec\theta\tan\theta$

O

$$\int_0^3 \frac{x^5}{\sqrt{1-x^2}}\,dx$$

x = 1 is prob ✓

$\boxed{x = \sin\theta}$ $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$dx = \cos\theta\, d\theta$

$\theta = \sin^{-1} x$

$\sin^{-1}(3)$ No such animal.

$\int x^2 \sin x \, dx$

$u = x^2$

$du =$

	u	dv
	x^2	$\sin x$
$+$	$2x$	$-\cos x$
$-$	2	$-\sin x$
$+$		$\cos x$

$uv \quad - \int v \, du$

$= -x^2 \cos x + 2x \sin x + 2\cos x + C$

$\int e^x \sin x \, dx$

	u	dv
$+$	e^x	$\sin x$
$-$	e^x	$-\cos x$
$+$	e^x	$-\sin x$
$-$	e^x	$\cos x$
	e^x	$\sin x$

$= -e^x \cos x + e^x \sin x$

$+ e^x \cos x - e^x \sin x + \cdots$

Don't see tabular helping, here.

Just $uv - \int v \, du$ twice & do algebra.

§9.1 skim it.

§9.2 Integrating Factor for Linear Diff. eq's.