$$\Delta y \approx dy = f'(x)\,dx$$

Nice for finding the amount of paint to cover the sides of a cylindrical water tank with a coat that's $\frac{1}{2}''$ thick. $R = 10'$, $H = 15'$

Volume of Tank:

$$\underline{V = \pi R^2 H}$$

Come back to this....

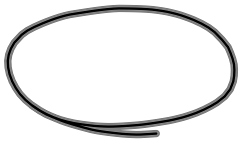
View as change in volume question

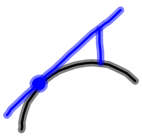
H is held constant

$$\boxed{V = 15\pi R^2}$$

$$\Delta R = \left(\tfrac{1}{2}\text{ in}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right) = \frac{1}{24}\text{ ft}$$

$$\frac{dV}{dR} = 30\pi R$$





$$dV \approx \Delta V = 30\pi R\,dR = 30\pi R\,\Delta R$$

$$= 30\pi(10)\left(\tfrac{1}{24}\right)$$

$$= \frac{300}{24}\pi = \frac{150}{12}\pi = \frac{75}{6}\pi = \frac{25}{2}\pi\text{ ft}^3$$

$\approx 39.2699\text{ ft}^3$ of paint.

over- or under-estimate?

UNDER-ESTIMATE

$V = V(R)$ is concave up.

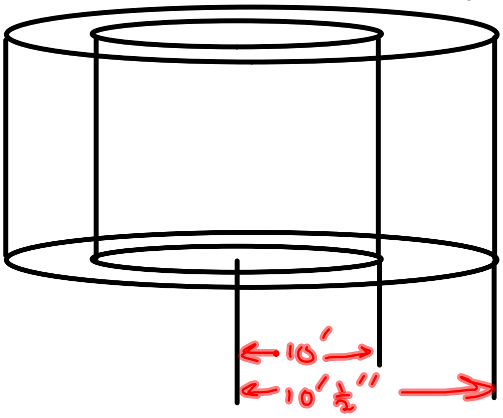
```
25/2*π
        39.26990817
```

Let's find the ACTUAL value for volume of paint needed

ΔV = Volume w/ new coat − Volume w/o new coat.

w/ : $10\text{ ft} + \frac{1}{24}\text{ ft} = 10.041\bar{6}\text{ ft}$

w/o : 10 ft

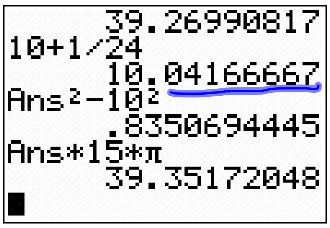


$V(10.041\bar{6}) - V(10)$

$= 15\pi(10.01\bar{6})^2 - 15\pi(10)^2$

$= 15\pi\left[10.01\bar{6}^2 - 10^2\right]$

$\approx 39.35172048\text{ ft}^3$



is slightly bigger than 39.26990817, which we obtained using differentials

§2.6 cont'd

Marginal Cost/Revenue/Profit

Marginal cost is the cost of producing one additional item.

$C(x)$ = cost function.

Marginal cost at a level of production of $x=100$ is

$C(101) - C(100)$ = cost of 101^{st} item.

$$\frac{C(101)-C(100)}{101-100} = \frac{C(101)-C(100)}{1} = C(101)-C(100)$$

$\approx C'(100) \approx$ Marginal Cost @ $x=100$.

we'll be comparing

$$\Delta C = \frac{\Delta C}{\Delta x} \approx C'(x)$$

If $\Delta x = 1$, which it IS in marginal cost discussions.

2.5, 2.6 Monday

2.3, 2.4 Take-Home FRIDAY NOON

2.7 Wednesday

$$\frac{d}{dx}\left[f(x)^{13}\right] = 13f(x)^{12}\cdot f'(x)$$

$$\frac{d}{dx}\left[(x^2-3x+2)^{27}\right] = 27(x^2-3x+2)^{26}(2x-3)$$

$\boxed{E}$ $$\frac{d}{dx}\left[\left(x^2-6x+\frac{1}{x}\right)^3\right] = \frac{d}{dx}\left[(x^2-6x+x^{-1})^3\right]$$

$$= 3(x^2-6x+x^{-1})^2(2x-6-x^{-2})$$

$$(3x^2-2x+1)\left(x^{-\frac{1}{2}}-\frac{1}{\sqrt{x}}+x^{\frac{2}{3}}\right)^{6/13}$$

$$= (3x^2-2x+1)\left(x^{\frac{2}{3}}\right)^{\frac{6}{13}}$$

$$= (3x^2-2x+1)\left(x^{\frac{12}{39}}\right)$$

0

$$x^{-\frac{1}{2}}-\frac{1}{\sqrt{x}}$$

$$= x^{-\frac{1}{2}}-x^{-\frac{1}{2}} = 0$$

$$(3x^2-2x+1)\left(x^{\frac{2}{3}}\right)^{\frac{6}{13}}$$

$$\Longrightarrow f'(x) = \ldots$$

$$(6x-2)\left(x^{\frac{2}{3}}\right)^{\frac{6}{13}} + (3x^2-2x+1)\underbrace{\left(\frac{6}{13}\right)\left(x^{\frac{2}{3}}\right)^{-\frac{7}{13}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)}_{\text{Chain rule on } \left(x^{2/3}\right)^{6/13}}$$

$$-\frac{14}{39} - \frac{13}{39} = -\frac{27}{39}$$

$$= (6x-2)\left(x^{2/3}\right)^{\frac{6}{13}} + (3x^2-2x+1)\left(\frac{12}{39}x^{-\frac{14}{39}}\right)\left(x^{-\frac{1}{3}}\right)$$

$$= (6x-2)\left(x^{\frac{12}{39}}\right) + (3x^2-2x+1)\left(\frac{12}{39}x^{-\frac{27}{39}}\right)$$

$$f(x) = (3x^2-2x+1)\left(x^{\frac{12}{39}}\right) \Longrightarrow$$

$$f'(x) = (6x-2)x^{\underbrace{\tfrac{12}{39}}_{\frac{4}{13}}} + (3x^2-2x+1)\left(\underbrace{\tfrac{12}{39}}_{\frac{4}{13}}x^{\overbrace{-\tfrac{27}{39}}^{-\frac{9}{13}}}\right)$$

$$\frac{d}{dx}\left[y^2\right] = 2y\frac{dy}{dx} = 2yy'$$

$$\frac{d}{dx}\left[f(x)^2\right] = 2f(x)\,f'(x)$$

Chain Rule, Baby.

$$\frac{d}{dx}\left[(x^2-3x+2)^2\right] = 2(x^2-3x+2)(2x-3)$$

2.7

$x^2+y^2=1$

Assume y is implicitly a function of x. Find $\frac{dy}{dx}$.

$$\frac{d}{dx}\left[x^2+y^2=1\right]$$

$$\cancel{\left(\frac{\sqrt{3}}{2}\right)^2=\frac{3}{4}}$$

$$2x+2yy'=0$$

$$2yy'=-2x$$

$$y'=-\frac{x}{y}$$

Find an equation of the tangent to this curve at the point $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

$$\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2=\frac{3}{4}+\frac{1}{4}=1$$

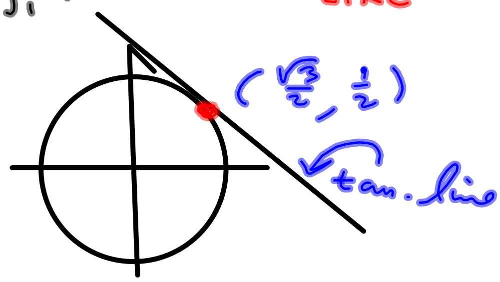
$$y'\Big|_{\substack{x=\frac{\sqrt{3}}{2}\\ y=\frac{1}{2}}}=-\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

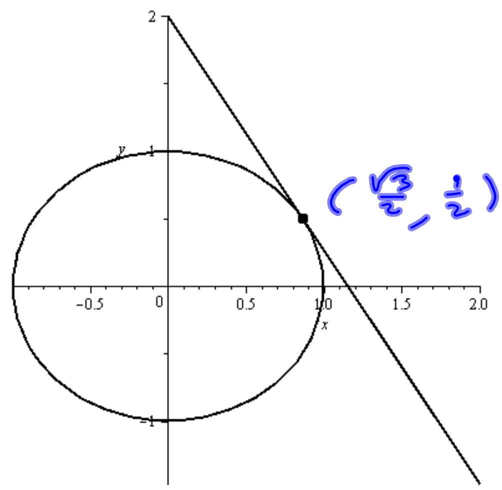
$$=-\frac{\sqrt{3}}{2}\cdot\frac{2}{1}=-\sqrt{3}=m_{tan}$$

$y=m(x-x_1)+y_1$

$y=y_1+m(x-x_1)$ ME LIKE

$$\boxed{y=-\sqrt{3}\left(x-\frac{\sqrt{3}}{2}\right)+\frac{1}{2}}$$





$y=-\sqrt{3}\left(x-\frac{\sqrt{3}}{2}\right)+\frac{1}{2}$

Pre-2.7 way:

$x^2+y^2=1$ Tangent to this curve @ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$y^2 = 1-x^2$

$\sqrt{y^2} = \sqrt{1-x^2}$ $\quad y=3 \quad \sqrt{y^2}=3$

$|y| = \sqrt{1-x^2}$ $\quad y=-3 \quad \sqrt{y^2}=3$

$y = \pm\sqrt{1-x^2} \longrightarrow \boxed{y=\sqrt{1-x^2}}$ is top $\frac{1}{2}$

$y = (1-x^2)^{\frac{1}{2}}$

$1-\frac{3}{4}=\frac{1}{4}$

$\sqrt{\frac{1}{4}}=\frac{1}{2}$

$\frac{dy}{dx} = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)$

$= \frac{-x}{\sqrt{1-x^2}} = \frac{-x}{y}$

So, $\left.\frac{dy}{dx}\right|_{x=\frac{\sqrt{3}}{2},\ y=\frac{1}{2}} = \frac{-\frac{\sqrt{3}}{2}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}}$

$= \frac{-\frac{\sqrt{3}}{2}}{\sqrt{\frac{1}{4}}} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2}\cdot\frac{2}{1} = -\sqrt{3}$

Find $\frac{dy}{dx}$:

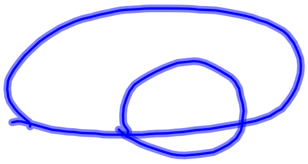
$$x^3y + x^2y^3 - y^2 = 77,898\pi$$

$$3x^2y + x^3y' + 2xy^3 + x^2 \cdot 3y^2y' - 2yy' = 0$$

$f = x^3 \qquad f' = 3x^2 = \frac{df}{dx}$

$g = y \qquad g' = y'$

$f'g + fg'$



$$x^3y' + 3x^2y^2y' - 2yy' = -3x^2y - 2xy^3$$

$$(x^3 + 3x^2y^2 - 2y)y' = -3x^2y - 2xy^3$$

$$\boxed{\frac{dy}{dx} = y' = \frac{-3x^2y - 2xy^3}{x^3 + 3x^2y^2 - 2y}}$$