Today Misc. Review

Haven't seen one like this:

$$\int_{-8}^{1} \frac{dx}{x^{1/3}} = \lim_{t\to 0^-} \int_{-8}^{t} \frac{dx}{x^{1/3}} + \lim_{t\to 0^+} \int_{t}^{1} \frac{dx}{x^{1/3}}$$

$$= \lim_{t\to 0^-} \frac{3}{2}x^{\frac{2}{3}}\Big]_{-8}^{t} + \lim_{t\to 0} \frac{3}{2}x^{2/3}\Big]_{t}^{1}$$

$$= \lim_{t\to 0^-}\left(\frac{3}{2}t^{\frac{2}{3}} - \frac{3}{2}(-8)^{\frac{2}{3}}\right) + \lim_{t\to 0}\left(\frac{3}{2} - \frac{3}{2}t^{\frac{2}{3}}\right)$$

$$= 0 - 6 + \frac{3}{2} - 0 = -\frac{9}{2}$$

If it converges, we can sort of ignore the technicalities.

$$\frac{3}{2}x^{\frac{2}{3}}\Big]_{-8}^{1} = -\frac{9}{2}$$

If you didn't catch it, for, say

$$\int_{-8}^{1} \frac{dx}{x^2} = -x^{-1}\Big]_{-8}^{1} = -\frac{1}{1} - \left(-\frac{1}{-8}\right)$$

$$= -1 - \frac{1}{8} = -\frac{9}{8}$$

We end up with garbage.

$\int$. The limit used in the 1st integral would have revealed this problem

$$\lim_{t \to 0^-} -\frac{1}{x}\Big]_{-8}^{t} \quad \cancel{\nexists}$$

$$+ \lim_{t \to 0^+} -\frac{1}{x}\Big]_{t}^{1} \quad \cancel{\nexists}$$

$$= \lim_{t \to 0^-} \left(-\frac{1}{t} - \left(-\frac{1}{-8}\right)\right) + \lim_{t \to 0^+} \left(-\frac{1}{1} - \left(-\frac{1}{t}\right)\right)$$

$\int x^3\sqrt{x^2+1}\,dx$ from §8.1, so we "don't know" trig. subst.

$x = a\tan\theta = \tan\theta \qquad dx = \sec^2\theta\, d\theta$

$$\int \tan^3\theta\sqrt{\tan^2\theta+1}\;\sec^2\theta\, d\theta$$

$$= \int \tan^3\theta\, \underline{\sec^3\theta}\, d\theta \quad \text{on the proper domain.}$$

$$= \int \tan^3\theta\,(\tan^2\theta+1)\sec\theta\, d\theta \quad \text{New p.}$$

$$\tan^3\theta\sec^3\theta = \sec^2\theta\,\underline{\tan^2\theta}\,(\sec\theta\tan\theta)$$

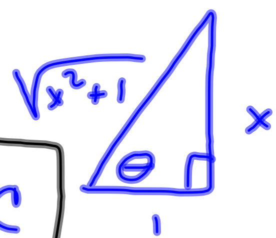
ODD powers of $\sec\theta$ & $\tan\theta$.

$$= \sec^2\theta(\sec^2\theta - 1)\boxed{\sec\theta\tan\theta}$$

$$= \sec^4\theta\cdot\sec\theta\tan\theta - \sec^2\theta\cdot\sec\theta\tan\theta$$

This gives $\int \sec^4\theta\sec\theta\tan\theta\, d\theta - \int\sec^2\theta\sec\theta\tan\theta\, d\theta$

$$= \frac{1}{5}\sec^5\theta - \frac{1}{3}\sec^3\theta + C$$



$$= \boxed{\frac{1}{5}\sqrt{x^2+1}^{\,5} - \frac{1}{3}\sqrt{x^2+1}^{\,3} + C}$$

$\sqrt{a^2+x^2}$ $\quad x = a\tan\theta \quad \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\sqrt{a^2-x^2}$ $\quad x = a\sin\theta \quad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sqrt{x^2-a^2}$ $\quad x = a\sec\theta \quad \theta \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

Domains: want these to be invertible substitutions. Make the trig functions 1-to-1

Feb 19-12:39 PM

$$\int x^3\sqrt{x^2+1}\,dx \qquad \frac{1}{5}\sqrt{x^2+1}^{\,5}+\frac{1}{3}\sqrt{x^2+1}^{\,3}+C$$

$u = x^3 \Longrightarrow du = 3x^2dx$

$dv = \sqrt{x^2+1}\,dx \Longrightarrow v =$ darn.

$u = \sqrt{x^2+1} = (x^2+1)^{\frac{1}{2}} \qquad du = \frac{1}{\cancel{2}}(x^2+1)^{-\frac{1}{2}}(\cancel{2}x)\,dx$

$dv = x^3dx \Longrightarrow v = \frac{1}{4}x^4$

$uv - \int v\,du = \frac{1}{4}x^4\sqrt{x^2+1} - \frac{1}{4}\int x^4\cdot x\cdot(x^2+1)^{-\frac{1}{2}}dx$

$-\frac{1}{4}\int \frac{x^5}{\sqrt{x^2+1}}\,dx$

$$\int x^3\sqrt{x^2+1}\,dx$$

$\cancel{u = x \quad du = dx}$

$\cancel{dv = x^2\sqrt{x^2+1}\,dx}$ Nowheah

$u = x^2 \qquad du = 2x\,dx$

$$dv = x\sqrt{x^2+1}\,dx = \frac{1}{2}\underbrace{(x^2+1)^{\frac{1}{2}}}_{r^{1/2}}\cdot\underbrace{2x\,dx}_{dr}$$

$$\Rightarrow v = \frac{1}{2}\,\frac{(x^2+1)^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{3}(x^2+1)^{3/2} = v$$

$$uv - \int v\,du = \frac{1}{3}x^2(x^2+1)^{\frac{3}{2}} - \frac{1}{3}\int (x^2+1)^{\frac{3}{2}}\cdot 2x\,dx$$

$$= \frac{1}{3}x^2(x^2+1)^{\frac{3}{2}} - \frac{1}{3}\cdot\frac{2}{5}(x^2+1)^{\frac{5}{2}} + C$$

$$= \frac{1}{3}x^2\sqrt{x^2+1}^{\,3} - \frac{2}{15}\sqrt{x^2+1}^{\,5} + C$$

$$= \frac{1}{5}\sqrt{x^2+1}^{\,5} - \frac{1}{3}\sqrt{x^2+1}^{\,3} + C$$

$\int x^2 \sin(x^3)\,dx$ Try Subst.

$\int x^2 \sin x\,dx$ IBP