Recall,

$\int_1^\infty \frac{dx}{x^2+1}$ converges, because it compares well with $\int_1^\infty \frac{dx}{x^2}$, which converges, by p-test.

$\frac{1}{x^2+1} < \frac{1}{x^2}$ Direct Comparison

But what about

$$\int_2^\infty \frac{dx}{x^2-1}$$

Don't have a nice Direct Comparison

WE DO have a LIMIT COMPARISON.

$$\lim_{x\to\infty} \frac{\frac{1}{x^2-1}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{x^2}{x^2-1} = 1$$

So they either both converge or both diverge

So $\int_{20}^\infty \frac{dx}{ax^2+bx+c}$ all converge.

the x^2 controls

(Assuming $\frac{1}{ax^2+bx+c}$ is cnt's on $[20,\infty)$)

§8.7 4, 11, 14, 22, 28, 41, 61

#s 35-64 - Just find if the $\int$ exists.

LIKE IVT questions.

Show that

Show that

§ 8.6 #19 estimate of error on [0,3]

$f(x) = (x+1)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x+1)^{-\frac{1}{2}}$

$f''(x) = -\frac{1}{4}(x+1)^{-\frac{3}{2}}$

$* \ f'''(x) = \frac{3}{8}(x+1)^{-5/2}$

$f^{(4)}(x) = -\frac{15}{16}(x+1)^{-7/2}$

$\boxed{h(x) = -\frac{15}{16}(x+1)^{-7/2}}$

$\boxed{h(0) = -\frac{15}{16}}$

$h(3) = -\frac{15}{16}\left(\frac{1}{128}\right)$

$= \boxed{-\frac{15}{2^{11}}}$

$h'(x) =$ something that's $\neq 0$ & undefined @ $x=-1 \notin [0,3]$

$M_{S'} \equiv \frac{15}{16}$

$|E_T|: M \geq \max_{x\in[0,3]} \{|f''(x)|\}$

$f''(x) = g(x) = -\frac{1}{4}(x+1)^{-3/2}$

$g(0) = -\frac{1}{4}$

$g(3) = -\frac{1}{4}(2)^{-3} = -\frac{1}{32}$

$g'(x) = \frac{3}{8}(x+1)^{-5/2}$

$= \frac{3}{8\sqrt{(x+1)^5}}$

So $\max_{x\in[0,3]}\{|f''(x)|\} = \frac{1}{4} \equiv M_T$

§ 8.3 # 10

§ 8.3 # 6 $\displaystyle\int_0^{\frac{1}{2\sqrt{2}}} \frac{2\,dx}{\sqrt{1-4x^2}}$

$$= \int_0^{\frac{1}{2\sqrt{2}}} \frac{2\,dx}{2\sqrt{\left(\frac{1}{2}\right)^2 - x^2}}$$

Let $x = \frac{1}{2}\sin\theta$

$$= \int_0^{\frac{1}{2\sqrt{2}}} \frac{2\,dx}{\sqrt{1^2 - (2x)^2}} = \int_0^{\frac{\pi}{4}} \frac{\cos\theta\,d\theta}{\cos\theta} = \int_0^{\frac{\pi}{4}} d\theta = \boxed{\frac{\pi}{4}}$$

Let $2x = \sin\theta$

$2\,dx = \cos\theta\,d\theta$

$$\boxed{\begin{aligned} \theta &= \arcsin(2x) \\ &\arcsin\left(2\cdot\tfrac{1}{2\sqrt{2}}\right) \\ &= \arcsin\left(\tfrac{1}{\sqrt{2}}\right) = \tfrac{\pi}{4} \\ &\arcsin(0) = 0 \end{aligned}}$$

$$\sqrt{1^2 - (2x)^2} = \sqrt{1^2 - \sin^2\theta}$$
$$= \sqrt{\cos^2\theta}$$
$$= |\cos\theta|$$
$$= \cos\theta \quad \text{on } [0, \tfrac{\pi}{4}]$$

without messing with limits of integration:

$$\int d\theta = \theta + C = \arcsin(2x) + C$$

This gives:

$$\arcsin(2x)\Big]_0^{\frac{1}{2\sqrt{2}}} = \arcsin\left(\tfrac{1}{\sqrt{2}}\right) - \arcsin(0)$$
$$= \frac{\pi}{4}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 6 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -1 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -1 & 4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 2 & 3 & -1 & 5 \\ 3 & 5 & -4 & 13 \end{array}\right]$$

Solve:

$$A + B - C = 3$$
$$2A + 3B - C = 5$$
$$3A + 5B - 4C = 13$$

Solve:

$A + B - C = 3 \rightarrow A = -B + C + 3$

$2A + 3B - C = 5 \rightarrow 2(-B+C+3) + 3B - C = 5$

$3A + 5B - 4C = 13 \rightarrow 3(-B+C+3) + 5B - 4C = 13$

$-2B + 2C + 6 + 3B - C = 5$

$B + C = -1$

$B = 3C - 1$

$\boxed{B = 8}$

All messed up.

$-3B + 3C + 9 + 5B - 4C = 13$

$2B - C = \cancel{13}\ 4$

$2(3C-1) - C = \cancel{13}\ 4$

$6C - 2 - C = \cancel{13}\ 4$

$5C = 15$

$\boxed{C = 3}$

$A = -8 + 3 + 3 = \boxed{-8 = A}$