$$= -\frac{1}{2}\int \frac{x-1}{x^2-4x+5}\,dx$$

$$\int \frac{x-1}{x^2-4x+5}\,dx = \int \frac{x-1}{(x-2)^2+1}\,dx$$

Let $u = x-2 \implies x = u+2$

Then $du = dx$

This gives

$$\int \frac{u+2-1}{u^2+1}\,du = \int \frac{u+1}{u^2+1}\,du$$

$v = u^2+1$

$dv = 2u\,du$

$$= \frac{1}{2}\int \frac{2u\,du}{u^2+1} + \int \frac{du}{u^2+1}$$

$$\frac{1}{2}\int \frac{dv}{v}$$

arctan . . .

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$$\frac{d^2y}{dx^2} = 2e^{-x}$$

$$\Rightarrow \frac{d}{dx}\left[\frac{dy}{dx}\right] = 2e^{-x}$$

$$\Rightarrow \int d\left[\frac{dy}{dx}\right] = \int 2e^{-x}\,dx$$

$y'(0) = 3$

$$y'(x) = \frac{dy}{dx} = -2e^{-x} + C$$

$$y'(0) = -2e^{-0} + C = 3$$

$$-2 + C = 3$$

$$C = 5 \longrightarrow$$

$$\frac{dy}{dx} = -2e^{-x} + 5$$

$$\int dy = \int (-2e^{-x} + 5)\,dx$$

$y(0) = 4$

$$y = 2e^{-x} + 5x + D$$

$$y(0) = 2 + D = 4$$

$$D = 2$$

$$\therefore \boxed{y = 2e^{-x} + 5x + 2}$$

$$y = \log_3(5xe^{2x})$$

$$= \log_3 5 + \log_3 x + \log_3 e^{2x}$$

$$\frac{d}{dx}\left[\frac{2}{\ln 3}x\right] = \frac{2}{\ln 3}$$

$$\frac{\ln(e^{2x})}{\ln 3} = \frac{2x}{\ln 3}$$

$$\Rightarrow y' = \frac{1}{\ln 3}\cdot\frac{1}{x} + \frac{1}{\ln 3}\,\frac{2e^{2x}}{e^{2x}}$$

$$= \qquad \frac{1}{\ln 3}\cdot 2$$

$$\frac{d}{dx}[\log_b x] = \frac{1}{\ln b}\cdot\frac{1}{x}$$

$$\frac{d}{dx}[\log_b f(x)] = \frac{1}{\ln b}\cdot\frac{f'(x)}{f(x)}$$

$$y = \log_3(5xe^{2x}) = \frac{1}{\ln 3}\ln(5xe^{2x})$$

$$\Rightarrow y' = \frac{1}{\ln 3}\left[\frac{5e^{2x} + 5x(2e^{2x})}{5xe^{2x}}\right] = \frac{1}{\ln 3}\left[\frac{1+2x}{x}\right]$$

$$= \frac{1}{(\ln 3)x} + \frac{2}{\ln 3}$$

3c If you forget $\frac{d}{dx}\left[b^{f(x)}\right] = (\ln b)\, b^{f(x)} f'(x)$

Logarithmic Differentiation is a sledgehammer.

$y = 5^{3x+2} \longrightarrow$

$\ln y = (3x+2)\ln 5 = (3\ln 5)\, x + 2\ln 5$

$\longrightarrow \quad \frac{y'}{y} = 3\ln 5$

$y' = (3\ln 5)\, y = (3\ln 5)\, 5^{3x+2}$

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$$y = (x^2+2x)^{x^2+2x}$$

$$\ln y = \ln\left((x^2+2x)^{x^2+2x}\right)$$

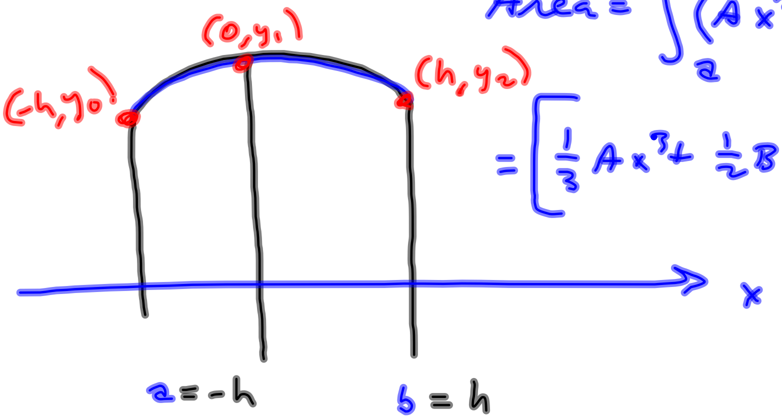
$$= (x^2+2x)\ln(x^2+2x)$$

$$\Rightarrow \frac{y'}{y} = (2x+2)\ln(x^2+2x) + \cancel{(x^2+2x)}\left(\frac{2x+2}{\cancel{x^2+2x}}\right)$$

$$\Rightarrow y' = \left((2x+2)\ln(x^2+2x) + (2x+2)\right)(x^2+2x)^{x^2+2x} \quad \text{STOP}$$

$$= (2x+2)\left[\ln(x^2+2x)+1\right](x^2+2x)^{x^2+2x}$$

§8.6 Simpson's



Area = $\int_a^b (Ax^2+Bx+C)\,dx$

$= \left[\frac{1}{3}Ax^3 + \frac{1}{2}Bx^2 + Cx\right]_a^b$

In the special case where $a=-h$ & $b=h$, we have

$\frac{1}{3}Ah^3 + \frac{1}{2}Bh^2 + Ch - \left[\frac{1}{3}A(-h)^3 + \frac{1}{2}B(-h)^2 + C(-h)\right]$

$= \frac{2}{3}Ah^3 + 2Ch = 2\left[\frac{Ah^3}{3} + Ch\right] = \frac{h}{3}\left[2Ah^2 + 6C\right]$

Now, since $(0, y_1)$ is on the graph of f,

we have $A(0)^3 + B(0)^2 + \boxed{C = y_1}$

$y_0 = A(-h)^2 + B(-h) + C = \underline{Ah^2 - Bh + y_1}$

$\Rightarrow \boxed{Ah^2 - Bh = y_0 - y_1}$

$y_1 = C$

$y_2 = Ah^2 + Bh + C = Ah^2 + Bh + y_1$

$$\boxed{Ah^2 + Bh = y_2 - y_1}$$

$\Longrightarrow 2Ah^2 = y_0 + y_2 - 2y_1$

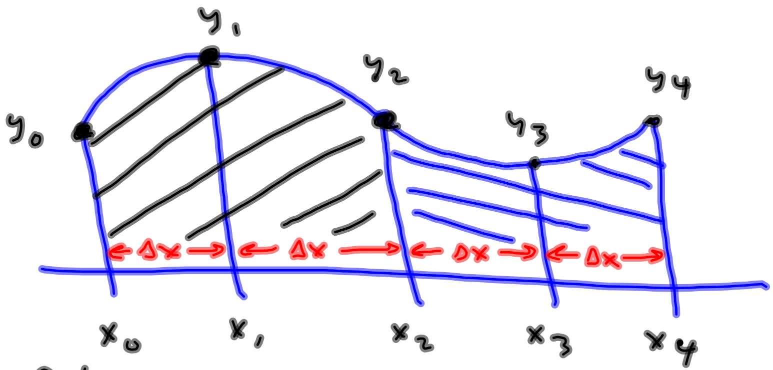
This gives Area $= \frac{h}{3}\left[2Ah^2 + 6C\right]$

$= \frac{h}{3}(y_0 + y_2 - 2y_1 + 6y_1)$

h is the Δx in the sequel.

$= \frac{h}{3}(y_0 + y_2 + 4y_1)$

$= \frac{h}{3}(y_0 + 4y_1 + y_2)$



2 of 'em

$$\frac{\Delta x}{3}\left[y_0 + 4y_1 + y_2 + y_2 + 4y_3 + y_4\right]$$

$= \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$

In general,

Area under $f(x)$ is given by Simpson's Rule, as follows:

$$S = \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 4y_{n-1} + y_n\right]$$

And you have to use $n = 2K$ (even).

$n = 4$ on homework.

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TRAP: Error Estimates.

If f'' cont^s & $|f''(x)| \le M$ on $[a,b]$

then $|E_T| \le \frac{M(b-a)^3}{12n^2}$

I'm interested in technique for finding M.

$f(x) = \sin\left(\frac{\pi}{3}x\right)$ on $[0, 2\pi]$

$f'(x) = \frac{\pi}{3}\cos\left(\frac{\pi}{3}x\right)$

$|f''(x)| = \left|-\frac{\pi^2}{9}\sin\left(\frac{\pi}{3}x\right)\right| = \frac{\pi^2}{9}\left|\sin\left(\frac{\pi}{3}x\right)\right|$

$\le \frac{\pi^2}{9}\cdot 1 = \boxed{\frac{\pi^2}{9} \equiv M}$

8.6 #s 7, 10, 13a

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