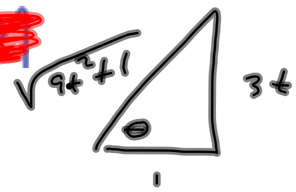
$$\int \frac{6\,dt}{(9t^2+1)^2} = \frac{1}{81}\int \frac{6\,dt}{(t^2+\frac{1}{9})^2} = \frac{1}{81}\int \frac{6\,dt}{((t^2+\frac{1}{9})^{\frac{1}{2}})}$$

$$t = \frac{1}{3}\tan\theta$$




$$dt = \frac{1}{3}\sec^2\theta\,d\theta$$

Gives

$$\frac{1}{81}\int \frac{6\cdot\frac{1}{3}\sec^2\theta\,d\theta}{(\frac{1}{9}\tan^2\theta + \frac{1}{9})^2} = \frac{1}{81}\int \frac{2\sec^2\theta\,d\theta}{(\frac{1}{9}\sec^2\theta)^2}$$

$$= \frac{81}{81}\int \frac{2\sec^2\theta\,d\theta}{\sec^4\theta} = 2\int \cos^2\theta\,d\theta$$

$$\int \frac{5\,dx}{\sqrt{25x^2-9}} \qquad x > \frac{3}{5}$$

$$\frac{dx}{\sqrt{x^2-a^2}}$$

26

$$\int \frac{x^2\,dx}{(x^2-1)^{5/2}}$$

36

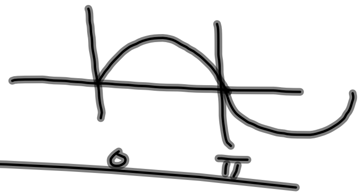
8.2 #24

$$\int_0^{\pi} \sqrt{1-\cos(2x)}\,dx$$

$$\sin^2 x = \frac{1-\cos(2x)}{2}$$

$$2\sin^2 x = 1-\cos(2x)$$

$$\sqrt{2\sin^2 x} = \sqrt{2}\,\sin x$$



$$\frac{Ax+B}{(x-2)^2+b^2} = \frac{Ax}{(x-2)^2+b^2} + \frac{B}{(x-2)^2+b^2}$$

$$\frac{-\frac{1}{2}x+\frac{1}{2}}{x^2-4x+5} = -\frac{1}{2}\cdot\frac{x-1}{x^2-4x+5}$$

$$\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left[\frac{2x-2}{x^2-4x+5}\right] = -\frac{1}{4}\left[\frac{2x-4+2}{x^2-4x+5}\right]$$

$u = x^2-4x+5$

$du = (2x-4)\,dx$

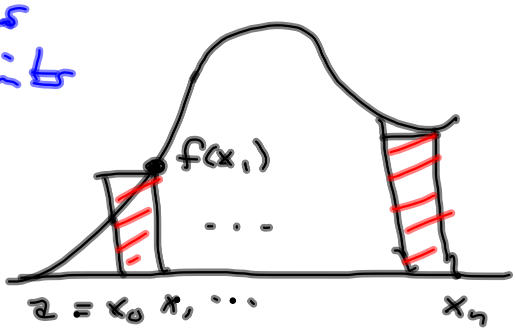
$$= -\frac{1}{4}\left(\frac{2x-4}{x^2-4x+5}\right) - \frac{1}{4}\left(\frac{2}{x^2-4x+5}\right)$$

$$\int\frac{dx}{x^2-4x+5} = \int\frac{dx}{(x-2)^2+1} = \int\frac{du}{u^2+1^2}$$

$u = x-2$

§8.6 Numerical Integration

Old Standby: Riemann Sums
Right end points




$$\Delta x = \frac{b-a}{n}$$

$$x_k = a + k\Delta x$$

$$\text{Area} \approx \sum_{k=1}^{n} f(x_k)\Delta x$$

Computers do these just fine.

Why?!

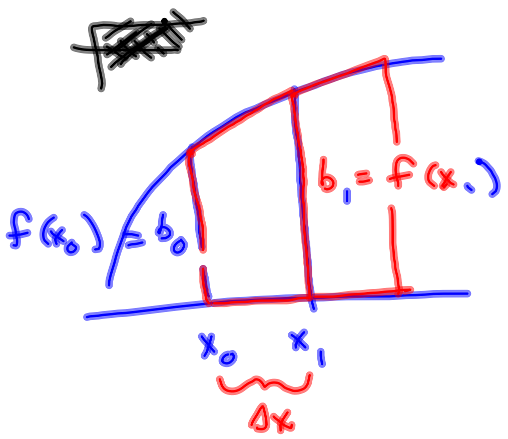
$$\int_0^5 e^{x^2}\,dx$$

has no closed-form antiderivative.
FTC fails us.

Trapezoid Rule & Simpson's Rule converge more rapidly to "the" value than Riemann.

Area of Trap: $\frac{1}{2}(b_1+b_2)h$

$$= \frac{1}{2}(f(x)+f(x))\Delta x$$




$\Delta x = \frac{b-a}{n}$, in all of these.

$$\text{Area} = \Delta x\left[\frac{1}{2}(f(x_0)+f(x_1)) + \frac{1}{2}(f(x_1)+f(x_2)) + \cdots + \frac{1}{2}(f(x_{n-1})+f(x_n))\right]$$

$$= \frac{\Delta x}{2}\left[y_0+y_1+y_1+y_2+y_2+\cdots+y_{n-1}+y_{n-1}+y_n\right]$$

$$= \frac{\Delta x}{2}\left[y_0+2y_1+2y_2+\cdots+2y_{n-1}+y_n\right]$$

$$\cancel{= \frac{\Delta x}{2}\left[y_0+\sum_{k=1}^{n-1} 2y_k+y_n\right]}$$

#10 $\int_0^1 \sin(\pi t)\,dt$ $n=4$ 8.6 #s 7, 10, 13a.

$= \frac{1}{\pi}\int_0^1 \sin(\pi t)\,\pi\,dt$

$= \frac{1}{\pi}\left(-\cos(\pi t)\right)\Big]_0^1$

$= -\frac{1}{\pi}\left(\cos\pi - \cos 0\right)$

$= -\frac{1}{\pi}\left(-1-1\right) = \frac{2}{\pi}$

$\Delta x = \frac{1-0}{4} = \frac{1}{4}$

$y_0 = \sin(\pi(0)) = 0$

$y_1 = \sin(\pi(\frac{1}{4})) = \frac{1}{\sqrt{2}}$

$y_2 = \sin(\pi(\frac{1}{2})) = 1$

$y_3 = \sin(\pi(\frac{3}{4})) = \frac{1}{\sqrt{2}}$

$y_4 = \sin(\pi) = 0$

$T_4 = \frac{\frac{1}{4}}{2}\left[0 + \frac{2}{\sqrt{2}} + 2 + \frac{2}{\sqrt{2}} + 0\right]$

$= \frac{1}{8}\left[\frac{4}{\sqrt{2}} + 2\right] = \frac{1}{2\sqrt{2}} + \frac{1}{4} \approx \frac{\sqrt{2}+1}{4} \approx .6035$