§8.1 26, 32, 40, 54

Plus 2 in-class

$\boxed{3}$ $\frac{d}{dt}\left[\tan^{-1}\left(\sqrt{t-1}\right)\right]$

$f(x) = \tan x$

$f'(x) = \sec^2 x$

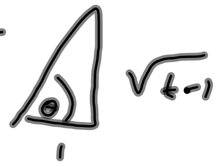
$$\boxed{\text{So } (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{\sec^2(\tan^{-1}(x))}}$$

Chain Rule:

$$\boxed{\frac{1}{\sec^2\left(\tan^{-1}\left(\sqrt{t-1}\right)\right)}} \cdot \boxed{\frac{1}{2\sqrt{t-1}}} = \frac{1}{2t\sqrt{t-1}}$$

$\frac{d}{d\sqrt{t-1}}\left[\text{original}\right] \cdot \frac{d}{dt}\sqrt{t-1}$

$\sqrt{\left(\sqrt{t-1}\right)^2 + 1^2} = \sqrt{t}$



$\sec^2\theta$

$= \left(\frac{\sqrt{t}}{1}\right)^2 = t$

$$\int \frac{dt}{t} = \ln|t| + C$$

$$\int \frac{dt}{t \pm a} = \ln|t \pm a| + C$$

$$\int \frac{ax+b}{(x-r_1)(x-r_2)}\,dx$$ Partial fracs gets us to here

what about when the denominator is an irreducible* quadratic factor?

*over the reals. *or over the rationals.

Special technique/assist:

① $$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C$$ (Meh)

② $$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$$

$\cancel{\frac{3\sqrt{2}}{5\sqrt{2}-11}}$ bleah.

$\boxed{E}$ How to manipulate:

$$\int \frac{dx}{x^2+4x-3} = \int \frac{dx}{(x+2)^2-7} = -\int \frac{dx}{\sqrt{7}^2-(x+2)^2}$$

x^2+4x+2^2-4-3

Let $u = x+2$. Then we have

$$= \int \frac{du}{\sqrt{7}^2-u^2}$$

$\sqrt{x^2} = |x|$

$(\sqrt{x})^2 = x$ (with the assumption that $x \geq 0$, else $\sqrt{x}$ wasn't real to start with)

$(\sqrt{7})^2 = 7$

$$\boxed{(x-(\underline{7+\sqrt{2}}))(x-(\underline{7-\sqrt{2}}))}$$

$$x^2-(7-\sqrt{2})x-(7+\sqrt{2})x+(7^2-\sqrt{2})^2$$

$$= x^2-7x+\sqrt{2}x-7x-\sqrt{2}x+47$$

$$= x^2-14x+47$$

Typically, we're going to break down $\frac{1}{x^2-a^2}$ into $\frac{A}{x-a}+\frac{B}{x+a}$ to integrate.

$$\int\frac{dx}{x^2-4x+8} = \int\frac{dx}{(x-2)^2+4} = \int\frac{du}{u^2+4}$$

$$x^2-4x+2^2-4+8$$

$$= (x-2)^2+4$$

$u=x-2$

$du=dx$

For handling irreducible quadratic factors.

The Basic skill

$$\int \frac{dx}{x^2-5x+6}$$

$$\frac{1}{x^2-5x+6} = \frac{A}{x-3} + \frac{B}{x-2} = \frac{1}{(x-3)(x-2)}$$

$$\Longrightarrow \quad 1 = A(x-2) + B(x-3) = Ax - 2A + Bx - 3B$$

$\boxed{M1}$

$0_x = Ax + Bx$

$0 = A + B$

$A = -B$

$1 = -2A - 3B$

$1 = -2(-B) - 3B$

$1 = -B$

$B = -1$

$A = +1$

$$\int \frac{dx}{x^2-5x+6} = \int \frac{dx}{x-3} - \int \frac{dx}{x-2}$$

$\boxed{M2}$

Let $x=2$

$1 = A(2-2) + B(2-3)$

$1 = -B$

$-1 = B$

Let $x=3$

$1 = A(3-2) + B(3-3)$

$1 = A$

Related to Heaviside "cover" technique.

Homework 8.4

1. $\int \frac{dx}{(x-2)(x+4)}$

$(x-(2+i))(x-(2-i))$
$= x^2-4x+5 = (x-2)^2+1$

2. $\int \frac{dx}{(x-2)^2(x-4)}$

4. $\int \frac{dx}{(x^2-4x+5)(x-3)}$

3. $\int \frac{dx}{(x^2-4x+5)}$

$\int 8.1$ Tomorrow
$\int 8.2, 8.3$ Wed.
$\int 8.4$ R

$x^2-4x+4-4+5$
$= (x-2)^2+1$
$u = x-2$
$= u^2+1$

$\int \frac{du}{u^2+1}$

Feb 11-12:54 PM

$$\int \frac{dx}{(x-2)^2(x-4)}$$

$$\frac{1}{(x-2)^2(x-4)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x-4}$$

$$A(x-2)(x-4) + B(x-4) + C(x-2)^2 = 1$$

$$A(x^2-6x+8) + Bx - 4B + C(x^2-4x+4) = 1$$

$$Ax^2 - 6Ax + 8A + Bx - 4B + Cx^2 - 4Cx + 4C = 1$$

$A + C = 0$

$A = -C$

$\boxed{A = -\frac{1}{4}}$

$-6A + B - 4C = 0$

$-6(-C) + B - 4C = 0$

$2C + B = 0$

$B = -2C$

$\boxed{B = -\frac{1}{2}}$

$8A - 4B + 4C = 1$

$8(-C) - 4B + 4C = 1$

$-4C - 4B = 1$

$-4C - 4(-2C) = 1$

$4C = 1$

$\boxed{C = \frac{1}{4}}$

$$\boxed{\frac{1}{(x-2)^2(x-4)} = \frac{A}{(x-2)^2} + \frac{B}{x-4}}$$

$$A(x-4) + B(x-2)^2 = 1$$

$$Ax - 4A + Bx^2 - 4Bx + 4B = 1$$

$$-4A + 4B = 1 \qquad A - 4B = 0 \qquad B = 0$$

Feb 12-12:37 PM

$$\int \sqrt{1+\sin x}\, dx$$

$$\int \frac{\sqrt{1-\sin^2 x}}{\sqrt{1-\sin x}}\, dx$$

$$= \int \frac{\sqrt{\cos^2 x}}{\sqrt{1-\sin x}}\, dx = \int \frac{\cos x\, dx}{\sqrt{1-\sin x}}$$

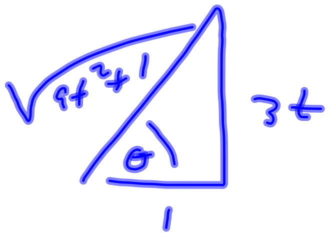
$$= -\int u^{-\frac{1}{2}} du$$

$$\int \frac{6\,dt}{(9t^2+1)^2} = \frac{1}{81}\int \frac{6\,dt}{(t^2+\frac{1}{9})^2} = \frac{1}{81}\int \frac{6\,dt}{((t^2+\frac{1}{9})^{\frac{1}{2}})^4}$$

$t = \frac{1}{3}\tan\theta$

$\tan\theta = 3t$

$\theta = \arctan(3t)$



Be nice to have a du for $u = 9t^2+1$.

Hmmmm

$u = (9t^2+1)^{-2}$ $\quad du = -2\,(9t^2+1)^{-3}(18t)\,dt$

$dv = dt$ $\quad v = t$