§8.1   26, 32, 40, 54

Plus 2 in-class

$\boxed{\text{B}}$ $\dfrac{d}{dt}\left[\tan^{-1}\left(\sqrt{t-1}\right)\right]$

$f(x) = \tan x$

$f'(x) = \sec^2 x$

So $\left(f^{-1}\right)'(x) = \dfrac{1}{f'(f^{-1}(x))} = \dfrac{1}{\sec^2(\tan^{-1}(x))}$

Chain Rule:

$\dfrac{1}{\sec^2(\tan^{-1}(\sqrt{t-1}))} \cdot \dfrac{1}{2\sqrt{t-1}} = \dfrac{1}{2t\sqrt{t-1}}$

$\dfrac{d}{d\sqrt{t-1}}\left[\text{original}\right] \cdot \dfrac{d}{dt}\sqrt{t-1}$

$\sqrt{\left(\sqrt{t-1}\right)^2 + 1^2} = \sqrt{t}$

$\sec^2\theta = \left(\dfrac{\sqrt{t}}{1}\right)^2 = t$

$$\int \frac{dt}{t} = \ln|t| + C$$

$$\int \frac{dt}{t \pm a} = \ln|t \pm a| + C$$

$$\int \frac{ax+b}{(x-r_1)(x-r_2)} \, dx \qquad \text{Partial fracs gets us to here}$$

what about when the denominator is an irreducible* quadratic factor?

* over the reals. * or over the rationals.

Special technique/assist:

① $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + C$ (Meh)

② $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$

$\frac{3\sqrt{2}}{5\sqrt{2}-11}$ bleah.

E) How to manipulate:

$$\int \frac{dx}{x^2+4x-3} = \int \frac{dx}{(x+2)^2-7} = -\int \frac{dx}{\sqrt{7}^2 - (x+2)^2}$$

x^2+4x+2^2-4-3

Let $u = x+2$. Then we have

$$\int \frac{du}{\sqrt{7}^2 - u^2}$$

$\sqrt{x^2} = |x|$

$(\sqrt{x})^2 = x$ (with the assumption that $x \ge 0$, else $\sqrt{x}$ wasn't real to start with)

$(\sqrt{7})^2 = 7$

$$(x - (7 + \sqrt{2}))(x - (7 - \sqrt{2}))$$

$$x^2 - (7 - \sqrt{2})x - (7 + \sqrt{2})x + (7^2 - \sqrt{2})^2$$

$$= x^2 - 7x + \sqrt{2}x - 7x - \sqrt{2}x + 47$$

$$= x^2 - 14x + 47$$

Typically, we're going to break down

$$\frac{1}{x^2 - a^2} \quad \text{into} \quad \frac{A}{x - a} + \frac{B}{x + a} \quad \text{to integrate.}$$

$$\int \frac{dx}{x^2 - 4x + 8} \quad = \quad \int \frac{dx}{(x-2)^2 + 4} = \int \frac{du}{u^2 + 4}$$

$$x^2 - 4x + 2^2 - 4 + 8 \qquad u = x - 2$$

$$= (x-2)^2 + 4 \qquad du = dx$$

For handling irreducible quadratic factors.

The __Basic__ skill

$$\int \frac{dx}{x^2-5x+6}$$

$$\frac{1}{x^2-5x+6} = \frac{A}{x-3} + \frac{B}{x-2} = \frac{1}{(x-3)(x-2)}$$

$$\implies \quad 1 = A(x-2) + B(x-3) = Ax - 2A + Bx - 3B$$

M1

$$0_x = Ax + Bx$$

$$0 = A + B$$

$$A = -B$$

$$1 = -2A - 3B$$

$$1 = -2(-B) - 3B$$

$$1 = -B$$

$$B = -1$$

$$A = +1$$

$$\int \frac{dx}{x^2-5x+6} = \int \frac{dx}{x-3} - \int \frac{dx}{x-2}$$

M2

let $x=2$

$$1 = A(2-2) + B(2-3)$$

$$1 = -B$$

$$-1 = B$$

let $x=3$

$$1 = A(3-2) + B(3-3)$$

$$1 = A$$

Related to Heaviside "cover" technique.

Homework 8.4

① $\int \dfrac{dx}{(x-2)(x+4)}$

$(x-(2+i))(x-(2-i))$

$= x^2 - 4x + 5 = (x-2)^2 + 1$

② $\int \dfrac{dx}{(x-2)^2(x-4)}$

④ $\int \dfrac{dx}{(x^2-4x+5)(x-3)}$

③ $\int \dfrac{dx}{(x^2-4x+5)}$

$\int 8.1$ Tomorrow

$\int 8.2, 8.3$ Wed.

$\int 8.4$ R