$$\int \cos^6 x \, dx$$

$$= \int \left(\frac{\cos(2x) + 1}{2} \right)^3 dx$$

$$= \frac{1}{8} \int \left(\overset{①}{\cos^3(2x)} + \overset{②}{3\cos^2(2x)} + \overset{③}{3\cos(2x)} + \overset{④}{1} \right) dx$$

$$= \frac{1}{8} \left[\int ① + \int ② + \int ③ + \int ④ \right]$$

① $\int \cos^3(2x) \, dx = \int (1 - \sin^2(2x)) \cos(2x) \, dx$

$$= \int \cos(2x) \, dx - \int \underset{u}{\sin^2(2x)} \underset{du}{\cos(2x) \, dx}$$

$$= \frac{1}{2}\sin(2x) - \frac{1}{3}\sin^3(2x) + C_1$$

② $\int \cos^2(2x) \, dx = \frac{1}{2} \int (\cos(4x) + 1) \, dx$

$$= \frac{1}{8}\sin(4x) + C$$

$$\int_0^1 x\sqrt{1-x}\,dx = -\int_1^0 (u-1)\sqrt{u}\,du$$

$u = 1-x$ $du = -dx$

$x = 0 \Rightarrow u = 1$

$x = 1 \Rightarrow u = 0$

---

§8.2 Methinks $\int \sec^3 x\,dx$ will come up, again, in CALC III.

SEE EXAMPLE.

$\frac{14}{2} = 7$

$$\sin^{14} x = \left(\frac{1-\cos(2x)}{2}\right)^{7}$$

The one left over is for the du part

$$\int \cos^7 x\,dx = \int (1-\sin^2 x)^3 \underbrace{\cos x\,dx}_{du}$$

for all the powers of sine that spit out.

$$\int \tan^n x\, dx = \int \tan^{n-2} x \tan^2 x\, dx$$

$$= \int \tan^{n-2} x \left(\sec^2 x - 1\right) dx$$

$$= \int \underbrace{\tan^{n-2} x}_{u^{n-2}} \underbrace{\sec^2 x\, dx}_{du}$$

$$- \int \tan^{n-2} x\, dx \text{, etc}$$

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$$\int \sin(3x)\cos(5x)\,dx$$

See Formulas:

① $\sin(mx)\sin(nx) =$

$$\frac{1}{2}\left[\cos((m-n)x) - \cos((m+n)x)\right]$$

② $\sin(mx)\cos(nx) =$

$$\frac{1}{2}\left[\sin((m-n)x) + \sin((m+n)x)\right]$$

③ $\cos(mx)\cos(nx) =$

$$\frac{1}{2}\left[\cos((m-n)x) + \cos((m+n)x)\right]$$

§ 8.2 #s 9, 14, 22, 24, 28, 44

§ 8.3 #s 6, 10, 17, 22, 23, 36

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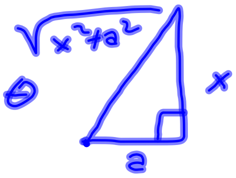
§8.3 For handling these forms:

$\sqrt{a^2+x^2}$ Let $x = a\tan\theta$

$dx = a\sec^2\theta \, d\theta$

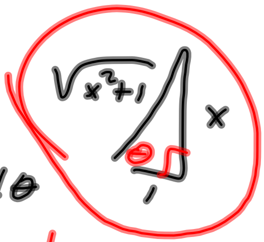
$\frac{x}{a} = \tan\theta$



$$\int \frac{dx}{(1+x^2)^{3/2}} = \int \frac{dx}{\left(\sqrt{1+x^2}\right)^3}$$

$x = \tan\theta$

$dx = \sec^2\theta \, d\theta$



we'll discuss the $|\sec\theta|$ issue, later.

$$= \int \frac{\sec^2\theta \, d\theta}{\left(\sqrt{1+\tan^2\theta}\right)^3} = \int \frac{\sec^2\theta \, d\theta}{\left(\sqrt{\sec^2\theta}\right)^3} =$$

$$= \int \frac{\sec^2\theta}{\sec^3\theta} \, d\theta = \int \frac{d\theta}{\sec\theta} = \int \cos\theta \, d\theta$$

$$= \sin\theta + C = \frac{x}{\sqrt{x^2+1}} + C$$

$$\frac{1}{2}\int \frac{2x}{\sqrt{x^2+1}} \, dx = \int \frac{du}{u^{1/2}}$$

$u = x^2+1$
$du = 2x \, dx$ } Sweet

When straight-up u-substitution works, it's gonna be easier than trig substitution.

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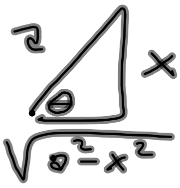
$\sqrt{a^2 - x^2}$ Let $x = a\sin\theta$ $\quad \frac{x}{a} = \sin\theta$

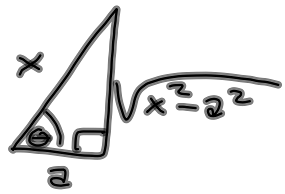
$dx = a\cos\theta\, d\theta$



$\sqrt{x^2 - a^2}$ Let $x = a\sec\theta$

$dx = a\sec\theta\tan\theta\, d\theta$

$\frac{x}{a} = \sec\theta$



$\sqrt{25x^2 - 4} = \sqrt{25\left(x^2 - \frac{4}{25}\right)}$

$= 5\sqrt{x^2 - \left(\frac{2}{5}\right)^2}$

ahhh $a = \frac{2}{5}$

§8.4

$$\int \frac{2x-5}{(x-3)(x+4)}\,dx$$

observe $\displaystyle\int \frac{2x-5}{(x-3)(x+4)}\,dx = \int \frac{A}{x-3}\,dx + \int \frac{B}{x+4}\,dx$

$= A\ln|x-3| + B\ln|x+4| + C$

$A\displaystyle\int \frac{dx}{x-3}$ $\quad u = x-3$, $du = dx$ $\quad = A\displaystyle\int \frac{du}{u} = \ln|u| + C$

If only we can find A & B!

Heaviside "cover" method is too tricksie 4 me.

$$\frac{2x-5}{(x-3)(x+4)} = \frac{A}{x-3} + \frac{B}{x+4} \Longrightarrow$$

$$2x-5 = A(x+4) + B(x-3) = Ax+4A+Bx-3B$$

M1

$x=-4$:

$2(-4)-5 = B(-4-3)$

$-13 = -7B$

$\frac{13}{7} = B$

$x=3$:

$1 = 7A$

$\frac{1}{7} = A$

This gives $\frac{1}{7}\int \frac{dx}{x-3} + \frac{13}{7}\int \frac{dx}{x+4}$

$$= \frac{1}{7}\ln|x-3| + \frac{13}{7}\ln|x+4| + C$$

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$$\frac{2x-5}{(x-3)^3(x+4)^2}$$

$$= \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{(x-3)^3} + \frac{D}{x+4} + \frac{E}{(x+4)^2}$$

Owie! Need a more general method:

M2 for last one:

$$2x-5 = A(x+4) + B(x-3) = Ax + 4A + Bx - 3B$$

$\Rightarrow$ $2x = Ax + Bx$ $-5 = 4A - 3B$

$2 = A + B$ $-5 = 4(2-B) - 3B$

SUBSTITUTION

$\Rightarrow A = 2 - B$ etc.

OR (Elimination) Matrix Methods

$$\begin{aligned} A + B &= 2 \\ 4A - 3B &= -5 \end{aligned} \longrightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 4 & -3 & -5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -7 & -13 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & \frac{13}{7} \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{7} \\ 0 & 1 & \frac{13}{7} \end{array}\right]$$