$$s(t) = \int_a^t \sqrt{1+\sinh^2 x}\,dx = \int_a^x \cosh x\,dx$$

7.1

find $f^{-1}(x)$ & $(f^{-1})'(x)$
&/or maybe $f^{-1}(7)$ or $(f^{-1})'(7)$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$\int \frac{3^{\sqrt{x}}}{\sqrt{x}}\,dx$$

Let $u = \sqrt{x} = x^{\frac{1}{2}}$

$du = \frac{1}{2}x^{-1/2}dx = \frac{1}{2\sqrt{x}}dx$

$\frac{du}{\frac{1}{2\sqrt{x}}} = 2\sqrt{x}\,du = dx$

e) $\int \frac{3^{\sqrt{x}}}{\sqrt{x}}dx$

$= \int \frac{3^u}{\cancel{\sqrt{x}}} 2\cancel{\sqrt{x}}\,du$

$= 2\int 3^u du = 2 \cdot \frac{1}{\ln 3} 3^u + C$

$3^u = e^{\ln(3^u)} = e^{(\ln 3)u}$

$\int e^{(\ln 3)u} du =$

$v = (\ln 3)u$

$dv = (\ln 3)du$

$= \frac{1}{\ln(3)} \int e^{(\ln 3)u} (\ln 3)\,du$

$= \frac{1}{\ln 3} \int e^v dv = \frac{1}{\ln 3} e^v + C = \frac{1}{\ln 3} e^{(\ln 3)u} + C$

$= \frac{1}{\ln 3} e^{\ln(3^u)} + C = \frac{1}{\ln 3} 3^u + C$

This gives $\boxed{\int \frac{3^{\sqrt{x}}}{\sqrt{x}}dx = \frac{2}{\ln 3} \cdot 3^{\sqrt{x}} + C}$

7.2 $\ln x = \int_1^x \frac{1}{t}\,dt$

$x > 0$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

$$\frac{d}{dx}[\ln(u(x))] = \underbrace{\frac{1}{u(x)}}_{\frac{d}{du}[\ln u]} \cdot \underbrace{u'(x)}_{\frac{du}{dx}}$$ (Chain Rule)

$$\frac{d}{dx}[\ln(u(x))] = \frac{u'(x)}{u(x)}$$

$$\int \frac{u'(x)\,dx}{u(x)} = \int \frac{du}{u} = \ln(u(x)) + C$$

Assume $u(x) > 0$

$$\frac{d}{dx}[\ln(-x)] = \frac{-1}{-x} = \frac{1}{x}$$ leads to

$$\int \frac{du}{u} = \int \frac{u'(x)}{u(x)}\,dx = \ln|u(x)| + C$$

$u(x) \neq 0$ extends the tool.

$$\frac{\sqrt{x}}{\sqrt{x}+5} = y$$

$$\frac{\sqrt{y}}{\sqrt{y}+5} = x$$

$$\sqrt{y} = x\sqrt{y} - 5x$$

$$\sqrt{y} - x\sqrt{y} = -5x$$

$$\sqrt{y}(1-x) = -5x$$

$$\sqrt{y} = \frac{-5x}{1-x} \Longrightarrow y = \left(\frac{-5x}{1-x}\right)^2 = \left(\frac{5x}{x-1}\right)^2$$

$$y = \left(\frac{5x}{x-1}\right)^2$$

$$(1-x)^4$$
$$=(x-1)^4$$

$$\boxed{y = \sqrt{\frac{x^2(x+2)}{(x-7)(25x+2)}}} = \left(\frac{x^2(x+2)}{(x-7)(25x+2)}\right)^{\frac{1}{2}} = \left(\frac{x^3+2x^2}{25x^2-173x-14}\right)^{\frac{1}{2}}$$

Find $\frac{dy}{dx}$

$$\ln y = \frac{1}{2}\ln\left(\frac{x^2(x+2)}{(x-7)(25x+2)}\right)$$

$$\ln y = \frac{1}{2}\left[2\ln(x) + \ln(x+2) - \ln(x-7) - \ln(25x+2)\right]$$

$$\frac{y'}{y} = \frac{1}{2}\left[\frac{2}{x} + \frac{1}{x+2} - \frac{1}{x-7} - \frac{25}{25x+2}\right]$$

$$y' = \frac{1}{2}\left[\frac{2}{x} + \frac{1}{x+2} - \frac{1}{x-7} - \frac{25}{25x+2}\right]\sqrt{\frac{x^2(x+2)}{(x-7)(25x+2)}}$$

$$y = \left(\frac{x^3+2x^2}{25x^2-173x-14}\right)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}\left(\frac{x^3+2x^2}{25x^2-173x-14}\right)^{-\frac{1}{2}}\left[\frac{(3x^2+4x)(25x^2-173x-14)-(x^3+2x^2)(50x-173)}{(25x^2-173x-14)^2}\right]$$

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