$$\lim \frac{3x+1}{x} - \frac{1}{\sin x} = \lim \frac{(3x+1)\sin x - x}{x\sin x}$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{3\sin x + (3x+1)\cos x - 1}{\sin x + x\cos x} = \lim_{x\to 0} \frac{3\sin x + 3x\cos x + \cos x - 1}{\sin x + x\cos x}$$

$$= \lim_{h\to 0} \frac{\overbrace{3\cos x + 3\cos x}^{6} - \overbrace{3x\sin x - \sin x}^{0}}{\underbrace{\cos x + \cos x}_{2} \underbrace{- x\sin x}_{0}} = 3$$

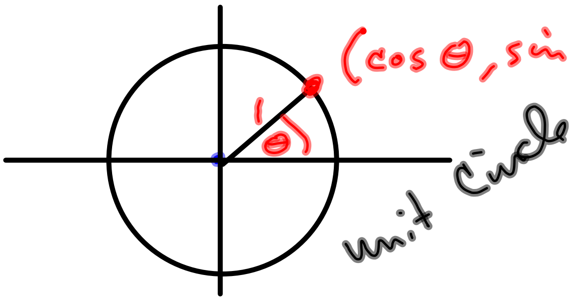
§7.7 Hyperbolic trig functions.

$\sinh x = \dfrac{e^x - e^{-x}}{2}$ $\qquad \dfrac{d}{dx}[\sinh x] = \dfrac{e^x + e^{-x}}{2} = \cosh x$

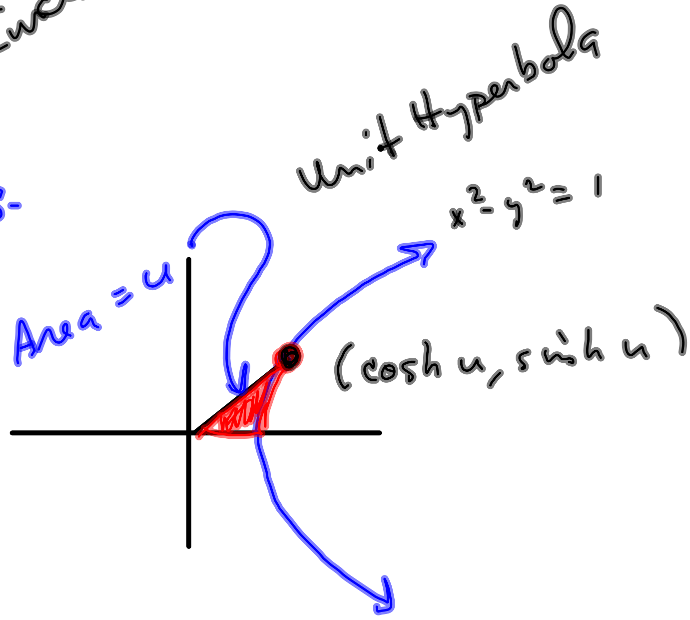
$\cosh x = \dfrac{e^x + e^{-x}}{2}$ $\qquad \dfrac{d}{dx}[\cosh x] = \dfrac{e^x - e^{-x}}{2} = \sinh x$

$\tanh x = \dfrac{\sinh x}{\cosh x}$ $\qquad \dfrac{d}{dx}[\tanh x] = \dfrac{\cosh^2 x - \sinh^2 x}{\cosh^2 x}$

$= \operatorname{sech}^2 x$



#s 83, 84, 86
to play & Discuss-



Worth knowing that there
IS a relationship between
sinh x & ln

$\sinh^{-1} x = \ln(x + \sqrt{x^2+1}) \quad -\infty < x < \infty$

See pg 422, 423

Jan 28-12:31 PM

f, g "eventually positive"

f & g grow at the same rate:

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = L \quad \text{for some } L \in \mathbb{R}$$

f is big-oh to g:

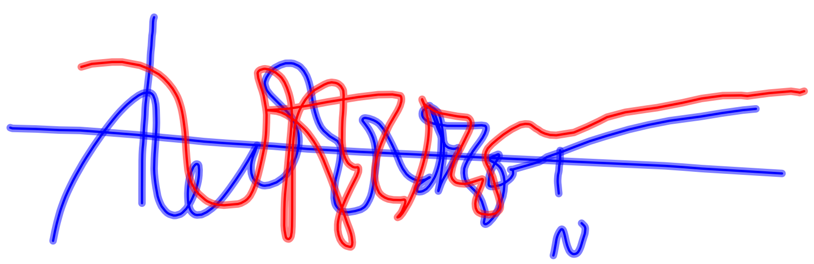
f grows at <u>most</u> as fast as g:

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} \leq M \quad \text{for some } M \in \mathbb{R}$$

f, g "eventually positive"

means $\underline{\exists\, N \in \mathbb{R}}$, such that

$f(x) > 0$ and $g(x) > 0$ for all $x > N$.



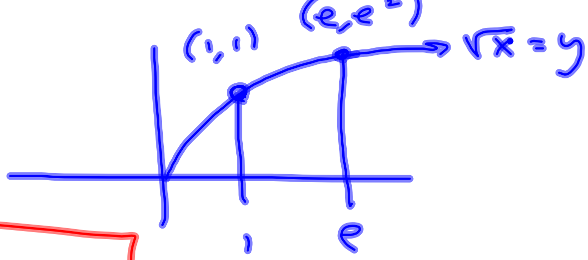
$$\lim_{x\to\infty} \frac{e^x}{x^2} \overset{L'H}{=} \lim_{x\to\infty} \frac{e^x}{2x} \overset{L'H}{=} \lim_{x\to\infty} \frac{e^x}{2} = \infty$$

$$x^2 = o(e^x)$$

$$\frac{x^2}{e^x} \xrightarrow{x\to\infty} 0$$

e^x <u>grows faster than</u> x^2

$$\boxed{\lim_{x\to\infty} \frac{e^x}{e^{x/2}}} = \lim_{x\to\infty} e^{\frac{x}{2}} = \lim_{x\to\infty} \left(e^{\frac{1}{2}}\right)^x = \infty$$



$$\boxed{\lim_{x\to\infty} \frac{e^x}{\frac{1}{2}e^x} = 2} \qquad e^x = O\left(\frac{e^x}{2}\right)$$

$$\lim_{x\to\infty} \frac{2^x}{3^x} = \lim_{x\to\infty} \left(\frac{2}{3}\right)^x = 0$$

$$2^x = o(3^x)$$

$$\lim_{x\to\infty} \frac{3^x-1}{2^x-1} = \lim_{x\to\infty} \frac{(\ln 3)\, 3^x}{(\ln 2)\, 2^x} = \lim_{x\to\infty} \frac{\ln 3}{\ln 2} \left(\frac{3}{2}\right)^x = \infty$$

$$\lim_{n\to\infty} \frac{e^n}{n!} = \lim_{n\to\infty} \frac{\overbrace{e\cdot e\cdot e\cdots e}^{n}}{n\cdot(n-1)\cdot(n-2)\cdots(3)(2)(1)}$$

I'll leave it open. Bonus

$$5! = 5\cdot4\cdot3\cdot2\cdot1$$

$$\frac{e^{n+1}}{(n+1)!} = \left(\frac{e}{n+1}\right)\frac{e^n}{n!}$$

§7.8 #s 9, 15
§7.7 #s 83, 84, 86

$$\lim_{x\to\infty} \frac{\ln(x-2)}{\ln(x+99)} = \lim_{x\to\infty} \frac{x+99}{x-2} = 1$$