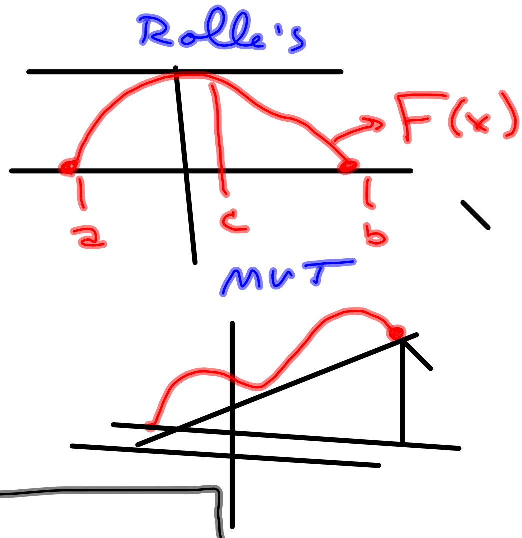
Cauchy's MVT



f, g cnt^s on $[a,b]$

f, g difb^l .. (a,b)

$g'(x) \neq 0$ on (a,b)

Then $\exists\ c \in (a,b) \ni$

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$$

$\boxed{\text{Pf}}$ Let $F(x) =$

$$f(x) - f(a) - \frac{f(b)-f(a)}{g(b)-g(a)}\left(g(x) - g(a)\right)$$

$F(a) = 0$

$F(b) = 0$

Apply Rolle's :

$\exists\ c \in (a,b) \ni F'(c) = 0$

$$F'(x) = f'(x) - \frac{f(b)-f(a)}{g(b)-g(a)}\, g'(x) \quad \&$$

$$\& \quad F'(c) = f'(c) - \frac{f(b)-f(a)}{g(b)-g(a)}\, g'(c) = 0$$

$$f'(c) = \frac{f(b)-f(a)}{g(b)-g(a)}\, g'(c)$$

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)} \quad \blacksquare$$

Only to help prove L'Hôpital's R...

$\frac{d}{dx}\left[\sec^{-1} x\right] = ?$

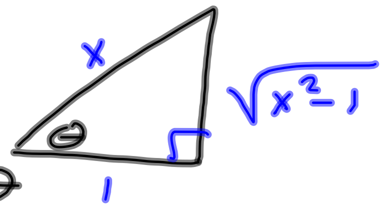
Pg 409, 410 gives a cool argument.

Here's another way:

$f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f^{-1}(x) = \sec^{-1} x = \operatorname{arcsec} x = \theta$

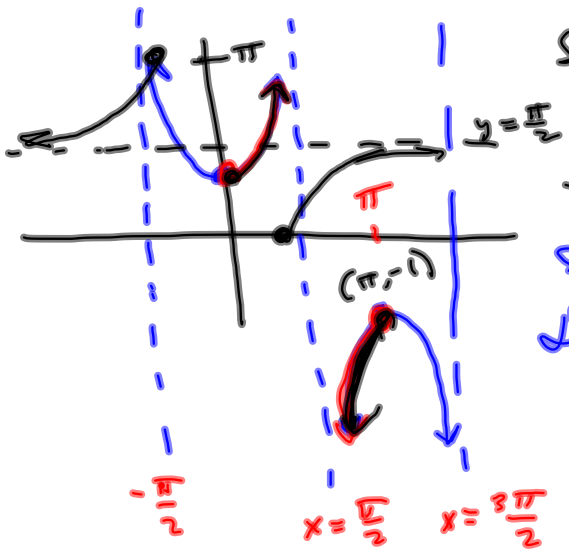


$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{\sec(\sec^{-1}x)\tan(\sec^{-1}x)}$$

$$= \frac{1}{x\sqrt{x^2-1}}$$

Actually, it SHOULD be $\frac{1}{|x|\sqrt{x^2-1}}$

Book gives good reason for



sec x

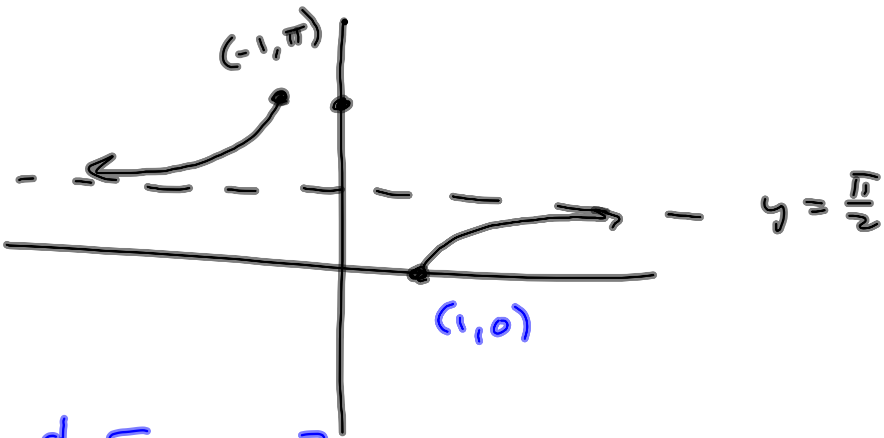
sec x is 1-to-1 on $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi] = \mathcal{D}$

$\mathcal{R} = [1, \infty) \cup (-\infty, -1]$

So, for $\sec^{-1} x$,

$\mathcal{D} = (-\infty, -1] \cup [1, \infty)$ &

$\mathcal{R} = [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$



$\frac{d}{dx}\left[\sec^{-1} x\right] > 0$ & $\frac{1}{|x|\sqrt{x^2-1}} > 0$

$\forall\ x \neq 0$

Book uses:

$\tan x = \pm\sqrt{\sec^2 x - 1}$

Test related to this:

Derive $\frac{d}{dx}[\sin^{-1}x]$ using, say, Theorem 1, §7.1

and starting with graph of $\sin x$,

like I did w/ $\sec x$, today.

See Table 7.4, pg 411
7.3

These concepts HUGE when we do trig. substitution, later.

§8.3 coming soon.

Jan 24-12:56 PM