$$\frac{d^2y}{dx^2} = e^x \quad , \quad y(0) = 0, y'(0) = 1$$

$$\int du = u + c$$

$$\frac{d}{dx}\left[\frac{dy}{dx}\right] = e^x$$

$$\int d\left[\frac{dy}{dx}\right] = \int e^x \, dx$$

$$y'(x) = \frac{dy}{dx} = e^x + C$$

$$y'(0) = 1 = e^0 + C = 1 + C = 1 \Longrightarrow C = 0$$

$$\frac{dy}{dx} = e^x$$

$$\int dy = \int e^x \, dx$$

$$y = e^x + C_1$$

$$y(0) = e^0 + C_1 = 1 + C_1 = 0 \Longrightarrow$$

$$1 = -C_1$$

$$-1 = C_1$$

$$\Longrightarrow \boxed{y = e^x - 1}$$

Jan 23-12:08 PM

$y'' = e^x \qquad y(0) = 0,\ y'(0) = 1$

$\int y'' = \int e^x$ — abusing the notation

$y' = e^x + C$

$y'(0) = 1 + C = 1 \Longrightarrow C = 0$

$\int y' = \int e^x$ — abusing the notation

$y = e^x + C_1$

$y(0) = 1 + C_1 = 0 \Longrightarrow C_1 = -1$

So $y(x) = e^x - 1$

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$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin(x) + C = \sin^{-1}(x) + C$$

Proof

$$\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{d}{dx}\left[f^{-1}(x)\right]$$

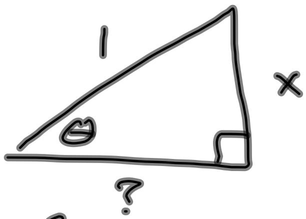
$$\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'(f^{-1}(x))} = \frac{1}{\cos(\sin^{-1}(x))}$$

$f(x) = \sin x$

$f'(x) = \cos x$

$f^{-1}(x) = \sin^{-1}(x)$

$\theta = \sin^{-1} x =$ angle whose sine is x.

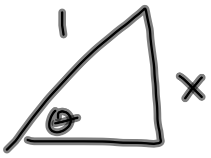


$$?^2 + x^2 = 1^2$$

$$?^2 = 1 - x^2$$

$$? = \pm\sqrt{1-x^2}$$

Assume ≥ 0 $\sqrt{1-x^2}$



$\cos\theta = \sqrt{1-x^2}$

So $(f^{-1})'(x) = \dfrac{1}{\sqrt{1-x^2}}$

#2e

§7.4 worksheet

$$\int f(x)\,f'(x)\,dx = \int u\,du = \frac{1}{2}u^2 + C$$

$u = f(x)$

$du = f'(x)\,dx$

$$\frac{1}{2}\int (x^4 - 3x^2)^{\frac{7}{11}}\, 2(2x^3 - 3x)\,dx = \int u^{7/11}\,du$$

$u = x^4 - 3x^2$

$du = (4x^3 - 6x)\,dx = 2\,(2x^3 - 3x)\,dx$

$$= \frac{11}{18}u^{\frac{18}{11}} + C \quad \text{etc.}$$

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$$\frac{dy}{dx} = y' = \frac{e^{2x-y}}{e^{x+y}} = e^{2x-y-(x+y)} = e^{x-2y}$$

$$= e^{x}e^{-2y}$$

$$\frac{1}{2^5} = 2^{-5}$$

$$\frac{e^{2x}e^{-y}}{e^{x}e^{y}} = \frac{e^{x}}{e^{2y}} = \frac{dy}{dx} \Rightarrow$$

$$e^{2y}dy = e^{x}dx$$

$$\frac{1}{2}e^{2y} = e^{x} + C_1 \qquad \frac{1}{2} \quad 2$$

$$e^{2y} = 2e^{x} + C_2$$

$$2y = \ln(2e^{x} + C_2)$$

$$y = \frac{1}{2}\ln(2e^{x} + C_2)$$

$$y = \ln\sqrt{2e^{x} + C}$$

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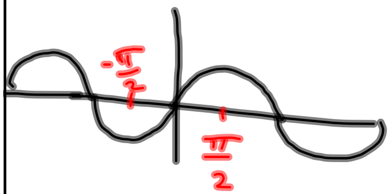
Trig Review for §7.6

$\sin^{-1}x$, $\tan^{-1}x$, $\cos^{-1}x$ is about all you need.

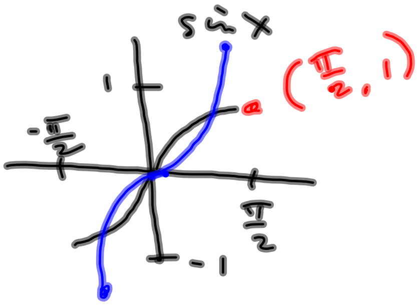
$\csc^{-1}x$, $\sec^{-1}x$, $\cot^{-1}x$ are HARDer and don't seem to come up that much.

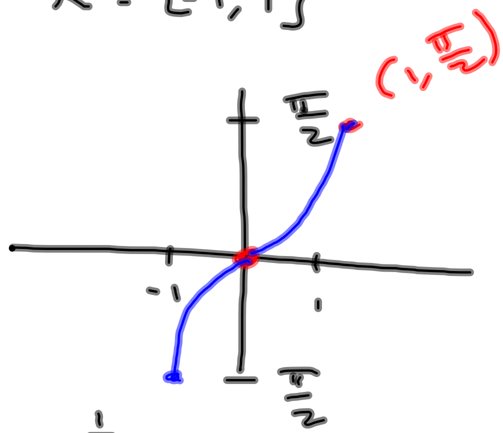
Restrictions on $\mathcal{D}$ to make 'em 1-to-1:



$\sin x$: $\mathcal{D} = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\mathcal{R} = [-1, 1]$





$$\frac{d}{dx}[\sin^{-1}x] = (1-x^2)^{-\frac{1}{2}} = \frac{1}{\sqrt{1-x^2}}$$

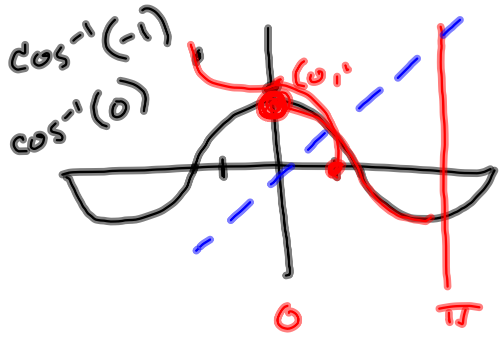
$$\frac{d^2}{dx}[\sin^{-1}x] = -\tfrac{1}{2}(1-x^2)^{-\frac{3}{2}}(-2x)$$

$$= \frac{x}{\sqrt{(1-x^2)^3}}$$

confirms concavity

$x>0$, $y''>0$ ✓

$x<0$, $y''<0$



$\cos x$: $\mathcal{D} = [0, \pi]$

$\mathcal{R} = [-1, 1]$

$\cos^{-1} x$: $\mathcal{D} = [-1, 1]$

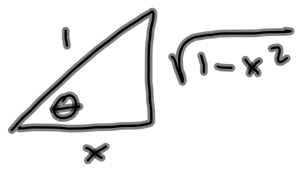
$\mathcal{R} = [0, \pi]$

$$\frac{d}{dx}\left[\cos^{-1} x\right] = \frac{d}{dx}\left[\arccos x\right]$$

$f(x) = \cos x$

$f'(x) = -\sin x$

$$\frac{1}{f'(f^{-1}(x))} = \frac{1}{-\sin(\underbrace{\cos^{-1} x}_{\theta})} = \frac{1}{-\sqrt{1-x^2}}$$



Up next time: $\tan^{-1}x$ & other stuff in §7.6