$$\frac{d}{dx}\left[\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}-3}\right]\Bigg|_{x=} = \frac{\frac{1}{2}x^{-\frac{1}{2}}(x^{\frac{1}{2}}-3) - x^{\frac{1}{2}}(\frac{1}{2}x^{-\frac{1}{2}})}{(x^{\frac{1}{2}}-3)^2}$$

$$\frac{\sqrt{x}}{\sqrt{x}-3} = 2$$

$$= \qquad \frac{1}{f'(f^{-1}(3))}$$

Methods:

① Solve $\frac{\sqrt{x}}{\sqrt{x}-3} = 2$

$f(x) = 3$ to

② $f^{-1}(2)$ from $f^{-1}(x)$. want $f^{-1}(3)$

$$\frac{(x-1)(x+2)(x-7)}{}$$

$$f^{-1}(f(x)) = f^{-1}(3)$$

$$x = f^{-1}(3)$$

$$e = \lim_{x \to 0} (1+x)^{\frac{1}{x}}$$

$\boxed{Pf}$ Let $f(x) = \ln(x)$. Then $f'(x) = \frac{1}{x}$

& $f'(1) = \frac{1}{1} = 1$

$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{\ln(1+h) - \ln(1)}{h}$$

$$= \lim_{x \to 0} \frac{\ln(1+x)}{x} = \lim_{x \to 0} \left(\frac{1}{x} \ln(1+x) \right)$$

$$= \lim_{x \to 0} \ln\left((1+x)^{\frac{1}{x}}\right) = e^{\ln\left(\lim_{x \to 0} \left((1+x)^{\frac{1}{x}}\right)\right)} = e^1$$

$$\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e \quad !$$

Re-label: Let $u = \frac{1}{x}$

$$\lim_{u \to \infty} \left(1 + \frac{1}{u}\right)^u = e$$

Compound interest: r = interest rate
m = # of periods per year
t = # of years
P = Principal
A = Future Value

FACT: $A(t) = P\left(1 + \frac{r}{m}\right)^{mt}$

Continuous Compounding

$$A(t) = Pe^{rt} = \lim_{m \to \infty} P\left(1 + \frac{r}{m}\right)^{mt}$$

$$\left(1 + \frac{r}{m}\right)^{mt} = \left(1 + \frac{r}{m}\right)^{\frac{m}{r} \cdot rt}$$

$$= \left(1 + \frac{r}{m}\right)^{\frac{1}{\frac{r}{m}} \cdot rt} = \left(\left(1 + \frac{r}{m}\right)^{\frac{1}{\frac{r}{m}}}\right)^{rt}$$

$$\xrightarrow{\frac{r}{m} \to 0} e^{rt}$$

Daily & Continuous are very close.

$$\int 3^x\,dx = \frac{1}{\ln 3}\cdot 3^x + C \qquad \frac{d}{dx}\left[3^x\right] = \ln 3 \cdot 3^x$$

$$\boxed{e^{\ln(3^x)} = 3^x}$$

$$\frac{d}{dx}\left[e^{\ln(3^x)}\right] = \frac{d}{dx}\left[e^{(\ln 3)x}\right] = (\ln 3)\,e^{(\ln 3)x}$$

$$= \ln 3 \cdot 3^x$$

Let $u = (\ln 3)x$

$du = (\ln 3)\,dx$

$$\frac{1}{\ln 3}\int e^{\boxed{(\ln 3)x}}\,\ln 3\,dx$$

$$= \frac{1}{\ln 3}\,e^{(\ln 3)x} + C = \frac{1}{\ln 3}\cdot 3^x + C$$

$$\frac{d}{dx}\left[\log_a x\right] = ?$$

$$\frac{d}{dx}\left[\frac{\ln x}{\ln a}\right] = \frac{1}{\ln a}\frac{d}{dx}\left[\ln x\right] = \frac{1}{\ln a}\cdot\frac{1}{x} = \frac{1}{x\ln a}$$

$$= \frac{1}{(\ln a)x}$$

$$y = \log_a x \Longrightarrow$$

$$a^y = x$$

$$\ln(a^y) = \ln x$$

$$y\ln(a) = \ln x$$

$y = \frac{\ln x}{\ln a}$ is change-of-base

$$? \int \log_a x \, dx = ??$$

?

?

Jan 17-12:41 PM

§7.4

$y' = ky \Longrightarrow y = Ce^{kt}$

$y' = -K(H - H_s)$

$y' = y'(x)$

$$\frac{y'}{y} = K$$

$$\int \frac{y'dx}{y} = \int K\,dx$$

$$\int \frac{dy}{y} = \int K\,dx$$

$$\int \frac{1}{y}\,dy = \int K\,dx$$

C_2

$C_2 - C_1$

If we Assume $y > 0$

$$e^{\ln y} = e^{Kx + \hat{C}}$$

$$y = e^{Kx+\hat{C}} = e^{Kx}e^{\hat{C}} = e^{\hat{C}}e^{Kx} = Ce^{Kx}$$

just a constant.

$\hat{C} = C_2 - C_1$

$C = e^{\hat{C}}$

Pop Growth
Radioactive Decay $\Big\}$ $y' = ky$ All

The half-life of Millsium is 70 years
A sample has 1/3 of the original Mills-ium present. How old is it?

$A(t) = $ Amt present $\Longrightarrow$

$A(t) = Ce^{kt}$

$A(70) = \cancel{C}e^{70k} = \frac{1}{2}A(0) = \frac{1}{2}\cancel{C}$

Solve for k

$A(0) = Ce^{k\cdot 0} = Ce^0 = C$ $\qquad e^{70k} = \frac{1}{2}$

Once we have k, then we can easily solve $Ce^{kt} = \frac{1}{3}C$ for t