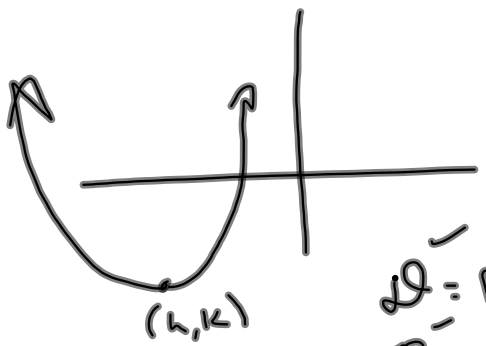
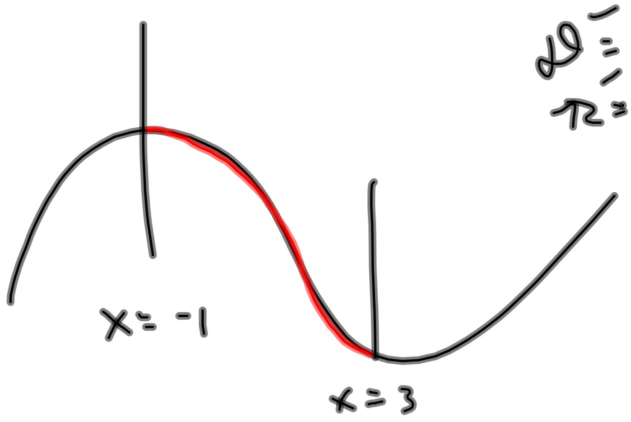


$\mathcal{D} = [h, \infty)$ 1st

$\mathcal{R} = [k, \infty)$

$\mathcal{D} = (-\infty, h]$ 2nd

$\mathcal{R} = [k, \infty)$



#3 You must use T7.1.1

Logarithmic Differentiation

$$y = f(x)$$

$$\ln y = \ln f(x) \quad \text{Legit if } f(x) > 0$$

$$\frac{y'}{y} = \frac{d}{dx}\left[\ln f(x)\right] \quad \text{easier than } \frac{d}{dx}\left[f(x)\right]$$

$$y' = \frac{d}{dx}\left[\ln f(x)\right] \cdot y \rightarrow f(x)$$

$$y = \frac{(x-1)^3(2x+1)(x^2+1)}{x^2-7}$$

$$\boxed{\begin{array}{l} \ln(ab) = \ln a + \ln b \\ \ln(a^b) = b \ln a \end{array}}$$

$$\ln y = \ln \left(\qquad \right)$$

$$= 3\ln(x-1) + \ln(2x+1) + \ln(x^2+1) - \ln(x^2-7)$$

$$\frac{y'}{y} = \left(3 \cdot \frac{1}{x-1} + \frac{2}{2x+1} + \frac{2x}{x^2+1} - \frac{2x}{x^2-7}\right)$$

$$y' = \left(\frac{3}{x-1} + \frac{2}{2x+1} + \frac{2x}{x^2+1} - \frac{2x}{x^2-7}\right)\left(\frac{(x-1)^3(2x+1)(x^2+1)}{x^2-7}\right)$$

$y' = \frac{d}{dx}[\ln x] = \frac{1}{x} \quad \forall\ x > 0$ Increasing.

$y'' = \frac{d}{dx}\left[\frac{1}{x}\right] = -\frac{1}{x^2} \quad \forall\ x > 0$ Concave Down.

inverse w.r.t. composition, not arithmetic

$\boxed{D}$ $\exp(x) = \ln^{-1}(x)$

BE Clear on context

Recall, e is the # such that $\ln(e) = 1$

and, (Now, we're in §7.3)

$$\ln(e) = 1$$

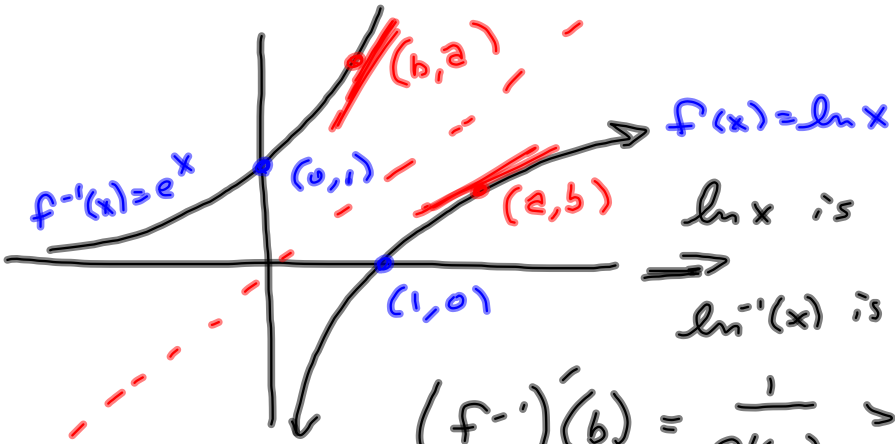
$$\ln^{-1}(\ln(e)) = \ln^{-1}(1) = \exp(1)$$

$$e = \exp(1)$$

$\ln x$ is increasing, with

$\mathcal{D}(\ln x) = (0,\infty) = \mathcal{R}(\ln^{-1}(x)) = \mathcal{R}(\exp(x)) = \mathcal{R}(e^x)$

$\mathcal{R}(\ln x) = (-\infty,\infty) = \mathcal{D}(e^x)$



$\ln x$ is increasing $\Longrightarrow$ $\ln^{-1}(x)$ is increasing!

$$(f^{-1})'(b) = \frac{1}{f'(a)} > 0 \text{ b/c } f'(a) > 0$$

$$= \frac{1}{f'(f^{-1}(b))}$$

The following is for r rational.

$r = \frac{p}{q}$

$a^{\frac{p}{q}} = \sqrt[q]{a^p}$

I don't think we need to worry about r rational/irrational.

Let's show that $\exp(x) = e^x$

we know that $\ln(e^r) = r\ln(e) = r$

So $\ln^{-1}(\ln(e^r)) = \ln^{-1}(r\ln(e)) = \ln^{-1}(r)$

$e^r = \exp(r)$ so they're the same.

① $\ln^{-1}(x) = \exp(x)$ ② e satisfies $\ln(e) = 1$

Jan 16-12:39 PM

$$\frac{d}{dx}\left[e^x\right] = ?$$

$$\frac{d}{dx}\left[\ln(u(x))\right] = \frac{u'(x)}{u(x)}$$

$$= \frac{1}{u(x)} \cdot \frac{du}{dx}$$

$$\ln(e^x) = x$$

$$\frac{d}{dx}\left[\ln(e^x)\right] = \frac{d}{dx}\left[x\right] = 1$$

$$\frac{1}{e^x} \cdot \frac{d}{dx}\left[e^x\right] = 1$$

$$\boxed{\frac{d}{dx}\left[e^x\right] = e^x}$$

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Chain Rule:

$$\frac{d}{dx}\left[e^{u(x)}\right] = e^{u(x)} \cdot u'(x) = e^{u(x)} \cdot \frac{du}{dx}$$
$$= \frac{du}{dx} e^{u(x)}$$

$\boxed{E}$ $\frac{d}{dx}\left[e^{3x}\right] = e^{3x} \cdot 3 = 3e^{3x}$

$$\frac{d}{dx}\left[e^{m x}\right] = m e^{m x}$$

$$e^{\ln b} = b$$

$$e^{\ln(a^x)} = a^x$$
$$= e^{x \ln(a)}$$
$$= e^{\ln(a) x}$$

$$\frac{d}{dx}\left[e^{\ln(a) x}\right] = e^{\ln(a) x} \cdot \ln a$$

$$\frac{d}{dx}\left[\ln(a) x\right] = \ln(a) \frac{d}{dx}[x]$$

$$= \ln(a) e^{\ln(a) x}$$
$$= \ln(a) e^{\ln(a^x)}$$
$$= \boxed{\ln(a)\, a^x = \frac{d}{dx}\left[a^x\right]}$$

$$a^x = e^{\ln(a^x)} = e^{\ln(a) x} \qquad = \ln(a) e^{x \ln(a)}$$

$$\frac{d}{dx}\left[a^x\right] = \ln(a) e^{\ln(a) x} = \ln(a) e^{\ln(a^x)} = \ln(a) a^x$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{1}{\ln(a)} a^x + C$$