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Find the inverse of

$f(x) = x^2 - 4x + 7$ on an "appropriate" domain.

$y^2 - 4y + 7 = x$

$y^2 - 4y + \underline{2^2} = x - 7 + \underline{4}$

$(y-2)^2 = x - 3$

$y - 2 = \pm\sqrt{x-3}$

$y = \pm\sqrt{x-3} + 2$

What's domain & range of $y = \sqrt{x-3} + 2$ ?

$\mathcal{D} = [3, \infty) = \mathcal{R}(f)$

$\boxed{\mathcal{R} = [2, \infty) = \mathcal{D}(f)}$

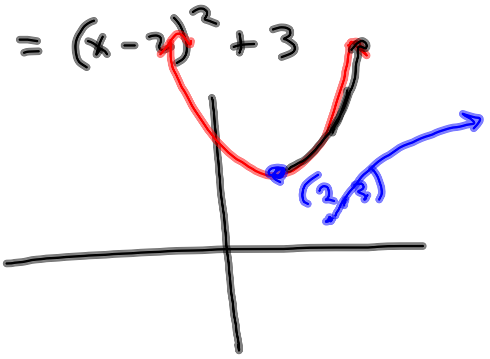
An appropriate restriction on $\mathcal{D}(f)$

$f(x) = x^2 - 4x + 7$

Another way to restrict the domain of f(x) to get it 1-to-1:

$= x^2 - 4x + 2^2 - 4 + 7$

$= (x-2)^2 + 3$




we chose $x \geq 2$
$y \geq 3$

$x^2 - 6x + 13$

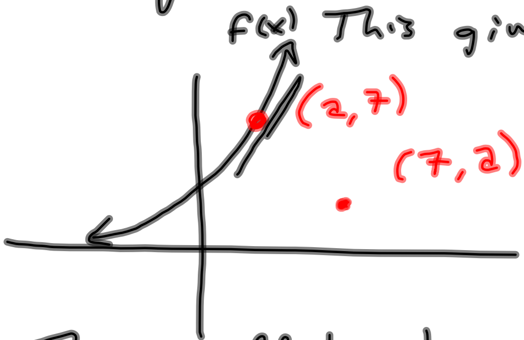
$1^{st}$ prob on coming worksheet.

A question-type:

$f^{-1}$ is hard. Find $(f^{-1})'(7)$

Technique: Solve $f(a)=7$.

This gives $(a,7)$ as your $(a,b)$




$$(f^{-1})'(b) = \frac{1}{f'(a)}$$

$$= \frac{1}{f'(f^{-1}(b))}$$

This will be done for a cubic

$f(x) = (x+1)(x-3)(x+2)$

but $f^{-1}(x)$ is impossible.

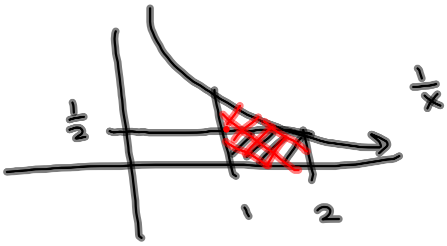
Rational Zeros is likely, here

S 7.1 # 32, 44

Last time, I said $\ln x$ grows without bound.
b/c $\int_1^\infty \frac{1}{x}\,dx$ doesn't converge.

This is Chapter 8!?

Argument in chapter 7:
Consider $\ln 2 = \int_1^2 \frac{1}{x}\,dx$

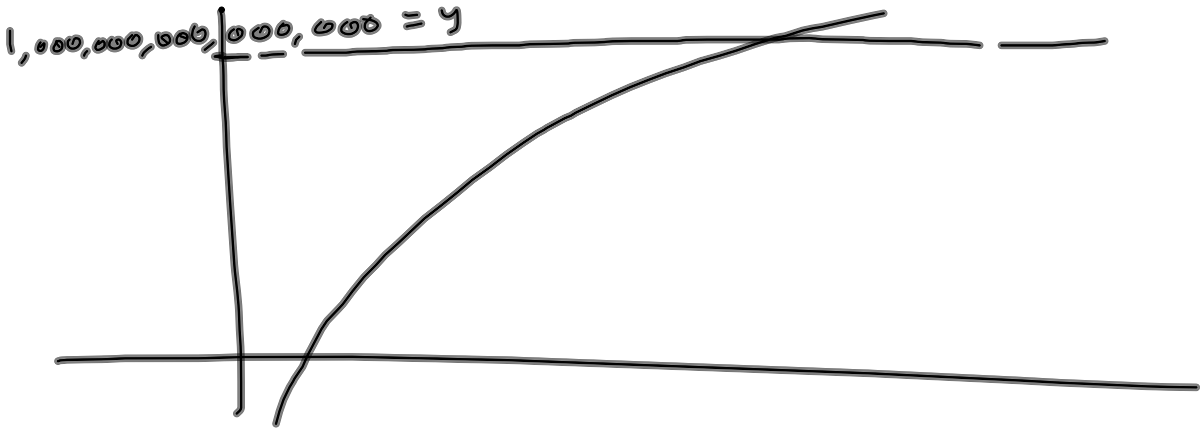



Area of rect is $\frac{1}{2}$
& $\int_1^2 \frac{1}{x}\,dx > \frac{1}{2}$
(shaded red)

$2^n$ can be made larger than <u>any</u> real number, just by taking $n$ sufficiently large.

So $\ln(2^n) = n\ln(2) > \frac{1}{2}n \xrightarrow{n \to \infty} \infty$

This proves that $\ln x$ grows without bound.




Jan 15-12:34 PM

See pg 371

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$\frac{d}{dx} \ln(bx) = \frac{1}{x} \quad !$$

By chain rule,

$$\frac{d}{dx}\left[\ln(u(x))\right] = \frac{d\ln(u)}{du} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\left[\ln(\cos x)\right] = \frac{1}{\cos x} \cdot (-\sin x)$$

Let $u = \cos x$ :

$$\frac{d}{dx}\left[\ln(\cos x)\right] = \frac{d}{dx}\left[\ln(u)\right]$$

$$= \frac{d}{du}\left[\ln u\right] \cdot \frac{d}{dx}\left[u\right]$$

$$\frac{d(\ln u)}{du} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\left[\ln(u(x))\right] = \frac{d\ln u}{du} \cdot \frac{du}{dx}$$

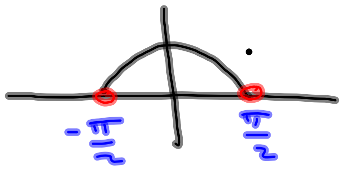
$$= \frac{1}{u} \cdot \frac{du}{dx}$$

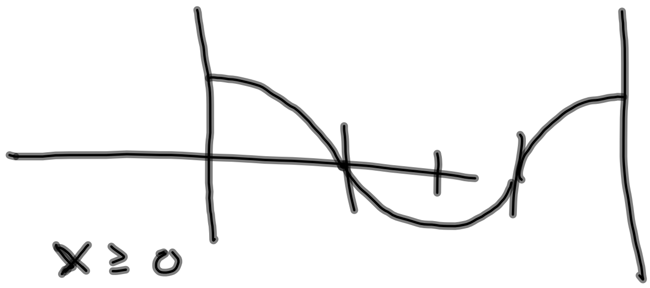
$$= \frac{1}{u} \cdot u'$$

$$= \frac{u'}{u} \quad !$$

Jan 15-12:48 PM

One thing about that last example.
Only good on a domain where $\cos x > 0$.
for instance on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



Since $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

$$\frac{d}{dx} \ln|x| = \begin{cases} \frac{1}{x} & x > 0 \\ \boxed{\frac{1}{x} \quad x < 0} \end{cases} = \frac{1}{x}, \; x \neq 0$$

If $x < 0$. Then chain rule sez

$$\frac{d}{dx} \ln|x| = \frac{d}{dx} \ln(-x) = \frac{1}{-x} \cdot (-1) = \frac{1}{x} \, !$$

$$\frac{d}{dx} \ln(-x) = \frac{-1}{-x} = \frac{1}{x}$$

$$\int \frac{1}{u}\,du = \ln|u| + c$$ Clint found this last semester.

This is the (a) key to

$$\boxed{\int \sec x\,dx}$$ trick involved.

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{-\sin x\,dx}{\cos x}$$

$u = \cos x$

$du = -\sin x\,dx$

$$= -\int \frac{du}{u} = -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

$$= \ln\left(|\cos x|^{-1}\right) = \ln|\sec x| + C$$

Jan 15-12:51 PM

Find $y'$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$y = \sqrt{\frac{t}{t+1}} = \left(\frac{t}{t+1}\right)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}\left(\frac{t}{t+1}\right)^{-\frac{1}{2}}\left(\frac{1(t+1) - t(1)}{(t+1)^2}\right)$$

Logarithmic Diff way:

$$\ln(y) = \ln\left(\sqrt{\frac{t}{t+1}}\right) = \frac{1}{2}\ln\left(\frac{t}{t+1}\right)$$

$\frac{d}{dx}$:

$$\frac{y'}{y} = \frac{1}{2}\left[\frac{\frac{t+1-t}{(t+1)^2}}{\frac{t}{t+1}}\right]$$

Jan 15-12:54 PM