http://www.harryzaims.com/202/202-spring-13

is where last fall's final solutions are.

Coming: $\ln x = \int_1^x \frac{1}{x}\,dx$

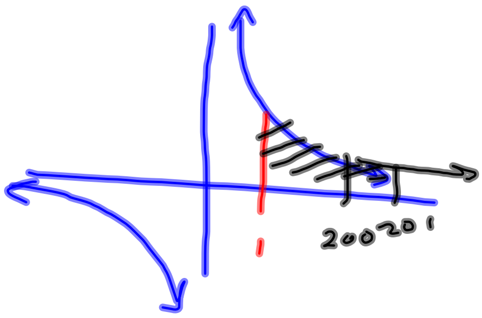
$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

$$\int_0^1 x^2\,dx = \frac{1}{3}x^3\Big]_0^1 = \frac{1}{3}$$

$$\int_1^0 x^2\,dx = -\int_0^1 x^2\,dx = -\frac{1}{3}$$

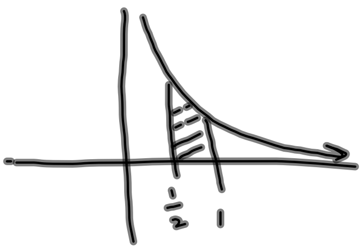
$\ln(1) =$

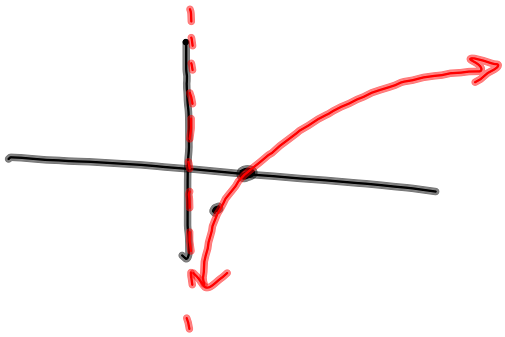
$$\int_1^1 \frac{1}{x}\,dx = 0$$



$\ln\left(\frac{1}{2}\right) =$ something less than zero

$$\int_1^{\frac{1}{2}} \frac{1}{x}\,dx = -\int_{\frac{1}{2}}^1 \frac{1}{x}\,dx$$





§7.1 f^{-1} & $(f^{-1})'$

$f^{-1} \neq \frac{1}{f}$ in a multiplicative sense.

f^{-1} is an inverse with respect to function composition.

Fed it x

$(f^{-1} \circ f)(x) =$ Identity function.

It spits out x

$f^{-1}(f(x)) = x$

$\boxed{E}$ $f(x) = x^3 \Rightarrow f^{-1}(x) = x^{\frac{1}{3}}$

Prove: $f(x) = x^3$. Let $g(x) = x^{\frac{1}{3}}$. Then

$(f \circ g)(x) = f(g(x)) = f(x^{\frac{1}{3}}) = (x^{\frac{1}{3}})^3 = x$

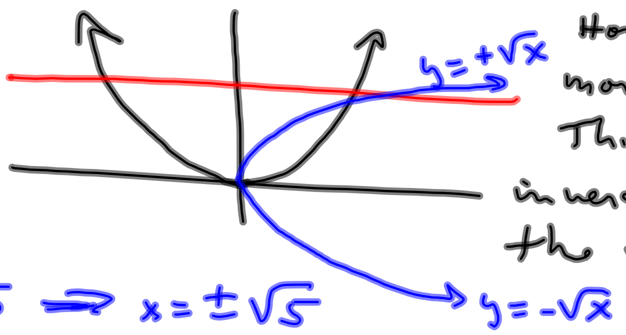
$\therefore g(x) = f^{-1}(x)$.

$\boxed{E}$ $f(x) = x^2$ does have an inverse RELATION, but NOT an inverse function.

$$x^3 = 5 \qquad\qquad x^2 = 5$$

$$x = 5^{\frac{1}{3}} \qquad\qquad x = 5^{\frac{1}{2}}$$

The trouble with x^2's inverse is x^2 isn't 1-to-1 on its domain.

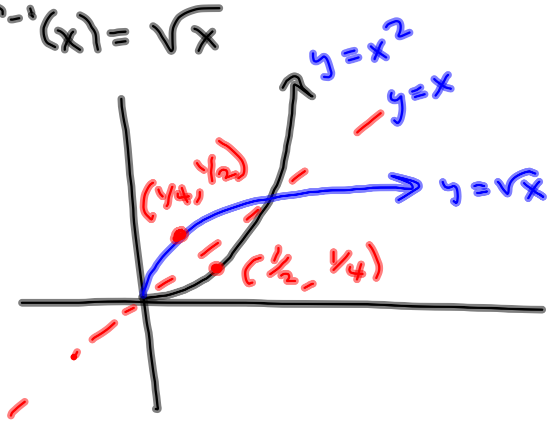


Horizontal line hits more than once. This will make the inverse relation fail the vertical line test.

$x^2 = 5 \Rightarrow x = \pm\sqrt{5}$

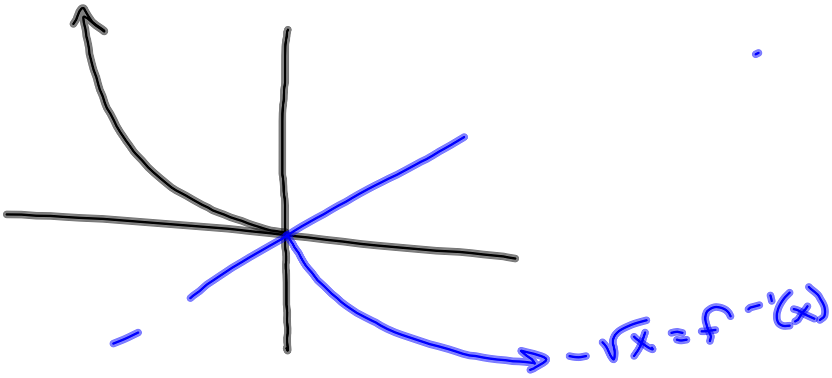
If we restrict $\mathcal{D} = [0, \infty)$, then

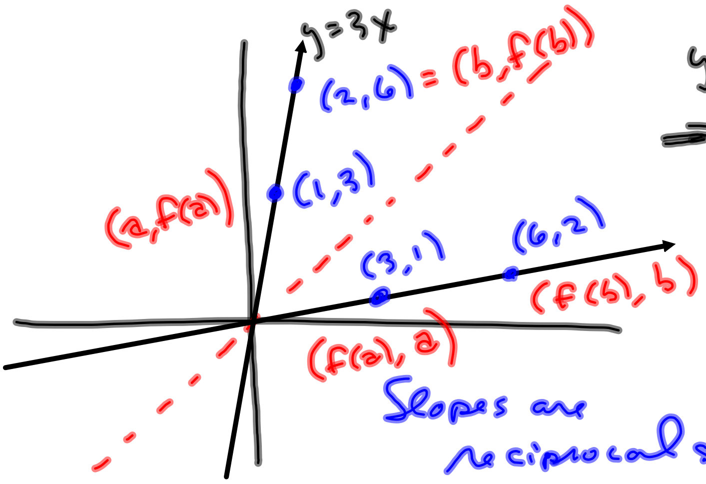
$f^{-1}(x) = \sqrt{x}$



If (a,b) on graph of $f(x)$, then (b,a)'s on graph of $f^{-1}(x)$.

x^2 for $x \le 0$





$y = 3x = f(x)$

$\Rightarrow f^{-1}(x) = \frac{1}{3}x$

$y = 3x$

$x = 3y \Rightarrow$

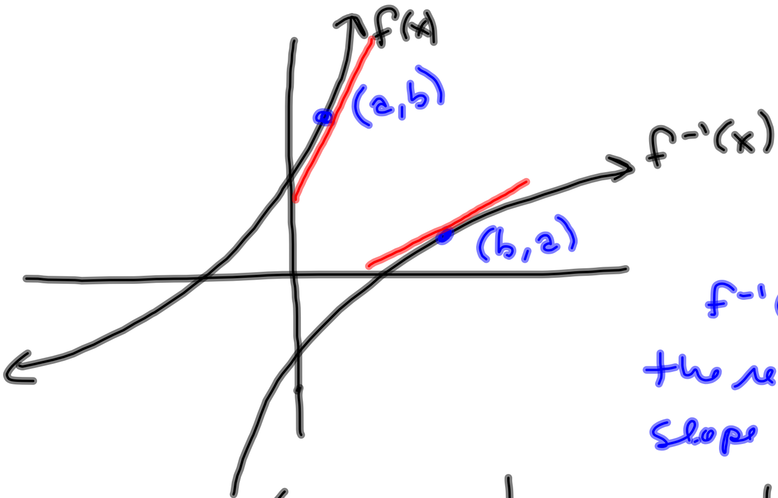
$3y = x$

$y = \frac{1}{3}x = f^{-1}(x)$

Slopes are reciprocals!

Slope of f is $\frac{f(b) - f(a)}{b - a} = \frac{6-3}{2-1}$

Slope of f^{-1} is $\frac{b-a}{f(b)-f(a)}$ is reciprocal.



The slope of $f^{-1}(x)$ at $x=b$ is the reciprocal of the slope of $f(x)$ at $x=a$ $= f^{-1}(b)$

$$(f^{-1})'(b) = \frac{1}{f'(a)} = \frac{1}{f'(f^{-1}(b))}$$

Handy when you have f^{-1} @ (b, a) and you have f', but $(f^{-1})'$ is hard.

Jan 14-12:50 PM