$a_1 = 1, \quad a_n = 3 - \frac{1}{a_{n-1}}$ or $a_{n+1} = 3 - \frac{1}{a_n}$

Prove a_n is increasing and bounded above by 3.

(1) $a_n > 1 \quad \forall \; n \geq 2$

(2) a_n is increasing

(3) a_n is bdd above by $y = 3$.

Pf

(1) $a_2 = 3 - \frac{1}{1} = 2 > 1$

Suppose $a_n > 1$ for some $n > 2$

Consider $a_{n+1} = 3 - \frac{1}{a_n} > 3 - \frac{1}{1} = 2 > 1$ $\blacksquare$

(2) NOTE $a_2 = 2 > 1 = a_1$

Suppose $a_{n+1} > a_n$

Consider $a_{n+2} = 3 - \frac{1}{a_{n+1}} > 3 - \frac{1}{a_n} = a_{n+1}$ $\blacksquare$

(3) By (1), we know $a_n > 1 \quad \forall \; n > 1$

Then $a_n = 3 - \frac{1}{a_{n-1}} < 3$ $\blacksquare$

$\therefore$ $3 - \frac{1}{a_n}$ is increasing & bdd above by $y = 3$,

and it converges. $\blacksquare$