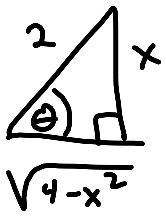
$\int$ 7.6 done thru #22-ish

Still nailing down this bad boy

$$\int x^3\sqrt{4x^2-x^4}\,dx$$

$$=\int x^4\sqrt{4-x^2}\,dx$$

Anybody who can do it without a painful trig subst., I'm all ears!



$x = 2\sin\theta \quad dx = 2\cos\theta\,d\theta$

$$=\int (2\sin\theta)^4\sqrt{4\cos^2\theta}\cdot 2\cos\theta\,d\theta$$

$$=2^4\int \sin^4\theta\cdot 2\cos\theta\cdot 2\cos\theta\,d\theta$$

$$=2^6\int\Big[\overbrace{\tfrac{1}{2}(1-\cos(2\theta))}^{\sin^2\theta}\Big]^2\cos^2\theta\,d\theta$$

$$=2^6\int \tfrac{1}{4}\left(1-2\cos(2\theta)+\cos^2(2\theta)\right)\left(\tfrac{1}{2}(1+\cos(2\theta)\right)d\theta$$

$$=\frac{2^6}{2^3}\int\left(1-2\cos(2\theta)+\cos^2(2\theta)\right)\left(1+\cos(2\theta)\right)d\theta$$

$$= 2^3 \int \Big[1 + \cos(2\theta) + 2\cos(2\theta) + 2\cos^2(2\theta) + \cos^2(2\theta) + \cos^3(2\theta) \Big]\, d\theta$$

$$= 2^3 \int d\theta + 2^3 \int 3\cos(2\theta)\, d\theta + 2^3 \int 3\cos^2(2\theta)\, d\theta + 2^3 \int \cos^3(2\theta)\, d\theta$$

$$= 8\theta + \frac{2^3 \cdot 3}{2} \sin(2\theta) + 2^3 \cdot 3 \int \frac{1}{2}\big(1 + \cos(4\theta)\big)\, d\theta$$

$$= 8\theta + 2^2 \cdot 3 \sin(2\theta) + \frac{2^3 \cdot 3}{2} \int d\theta + \frac{2^3 \cdot 3}{2 \cdot 4} \int \cos(4\theta) \cdot 4\, d\theta$$

$$= 20\theta + 12\sin(2\theta) + 3\sin(4\theta)$$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$$= 20\arcsin\left(\frac{x}{2}\right) + 12\left(2\sin\theta\cos(\theta)\right) +$$

$$+\ 3\left(2\sin(2\theta)\cos(2\theta)\right)$$

$$= 20\arcsin\left(\frac{x}{2}\right) + 24\left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right)$$

$$+\ 6\left[2\sin\theta\cos\theta\left(\cos^2\theta - \sin^2(\theta)\right)\right]$$

$$= 20\arcsin\left(\frac{x}{2}\right) + 6x\sqrt{4-x^2} + 12\left[\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2}\left(\frac{4-x^2}{4} - \frac{x^2}{4}\right)\right]$$

$$= 20\arcsin\left(\frac{x}{2}\right) + 6x\sqrt{4-x^2} + \frac{3}{4}\left[x\cdot\sqrt{4-x^2}\left(4-2x^2\right)\right]$$

$$\therefore \int_0^2 \text{stuff} = 20\arcsin(1) + 6(2)\cdot 0 + \frac{3}{4}\left[2\cdot 0\right]$$

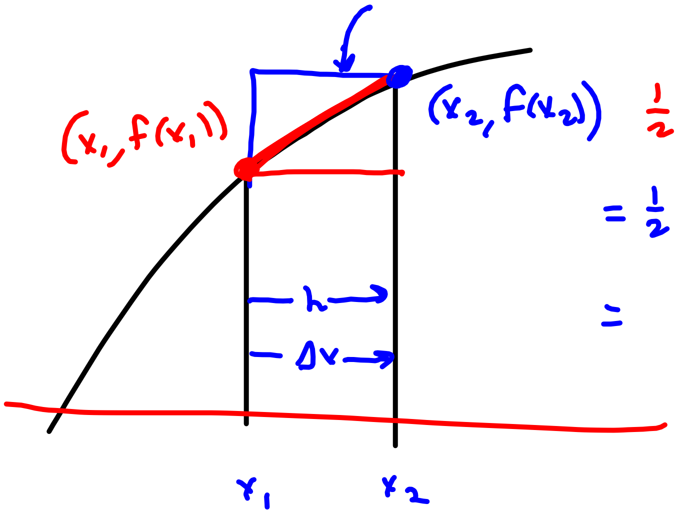
$$-\left[20\arcsin(0) + 6\cdot 0 + \frac{3}{4}\left[0\right]\right]$$

$$= \boxed{10\pi}\ ?\ \text{Should be } 2\pi$$

$=$

$\arcsin(1)$

§7.7 #s 7, 10, 25



$\frac{1}{2}(b_1+b_2)h$

$= \frac{1}{2}(f(x_1)+f(x_2))\Delta x$

$= \dfrac{f(x_1)\Delta x + f(x_2)\Delta x}{2}$

§7.6 #22

$$\int_0^2 x^3\sqrt{4x^2-x^4}\,dx = \frac{1}{2}\int_{0=u}^{4=4} x^2\sqrt{4u-u^2}\cdot 2x\,dx$$

Let $u = x^2$

$du = 2x\,dx$

$x=0 \Longrightarrow u = 0^2 = 0$

$x=2 \Longrightarrow u = 2^2 = 4$

$$= \frac{1}{2}\int_0^4 u\sqrt{4u-u^2}\,du$$

Formula #114 slays it, then

Much slicker than my method.

Nicholas & Scott