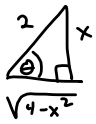
§7.6 done thru #22-ish

Still nailing down this bad boy

$$\int x^3\sqrt{4x^2-x^4}\,dx$$

$$= \int x^4\sqrt{4-x^2}\,dx$$

Anybody who can do it without a painful trig subst., I'm all ears!



$x = 2\sin\theta \quad dx = 2\cos\theta\,d\theta$

$$= \int (2\sin\theta)^4\sqrt{4\cos^2\theta}\cdot 2\cos\theta\,d\theta$$

$$= 2^4\int \sin^4\theta\cdot 2\cos\theta\cdot 2\cos\theta\,d\theta$$

$$= 2^6\int \left[\tfrac{1}{2}(1-\cos(2\theta))\right]^2\cos^2\theta\,d\theta \qquad (\sin^2\theta)$$

$$= 2^6\int \tfrac{1}{4}(1-2\cos(2\theta)+\cos^2(2\theta))\left(\tfrac{1}{2}(1+\cos(2\theta))\right)d\theta$$

$$= \frac{2^6}{2^3}\int (1-2\cos(2\theta)+\cos^2(2\theta))(1+\cos(2\theta))\,d\theta$$

$$= 2^3\int \Big[1+\cos(2\theta)+2\cos(2\theta)+2\cos^2(2\theta) + \cos^2(2\theta)+\cos^3(2\theta)\Big]d\theta$$

$$= 2^3\int d\theta + 2^3\int 3\cos(2\theta)\,d\theta + 2^3\int 3\cos^2(2\theta)\,d\theta + 2^3\int \cos^3(2\theta)\,d\theta$$

$$= 8\theta + \frac{2^3\cdot 3}{2}\sin(2\theta) + 2^3\cdot 3\int \tfrac{1}{2}(1+\cos(4\theta))\,d\theta$$

$$= 8\theta + 2^2\cdot 3\sin(2\theta) + \frac{2^3\cdot 3}{2}\int d\theta + \frac{2^3\cdot 3}{2\cdot 4}\int \cos(4\theta)\cdot 4\,d\theta$$

$$= 20\theta + 12\sin(2\theta) + 3\sin(4\theta) \qquad \cos(2\theta) = \cos^2\theta-\sin^2\theta$$

$$= 20\arcsin\left(\tfrac{x}{2}\right) + 12(2\sin\theta\cos\theta) + 3(2\sin(2\theta)\cos(2\theta))$$

$$= 20\arcsin\left(\tfrac{x}{2}\right) + 24\left(\tfrac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right) + 6\left[2\sin\theta\cos\theta(\cos^2\theta-\sin^2\theta)\right]$$

$$= 20\arcsin\left(\tfrac{x}{2}\right) + 6x\sqrt{4-x^2} + 12\left[\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2}\left(\frac{4-x^2}{4}-\frac{x^2}{4}\right)\right]$$

$$= 20\arcsin\left(\tfrac{x}{2}\right) + 6x\sqrt{4-x^2} + \frac{3}{4}\left[x\cdot\sqrt{4-x^2}\,(4-2x^2)\right]$$

$$\therefore \int_0^2 \text{stuff} = 20\arcsin(1) + 6(2)\cdot 0 + \tfrac{3}{4}[2\cdot 0] - \left[20\arcsin(0) + 6\cdot 0 + \tfrac{3}{4}[0]\right]$$

$$= \boxed{10\pi}$$? Should be 2π

$\arcsin(1)$