

§7.1 Theory & Exercises thru #47  
are posted in video & notes.

I'm interested in #47, today  
 $u, v$  are functions of  $x$ . Then

$$(uv)' = u'v + uv'$$

$$\frac{d}{dx}[uv] = \frac{d}{dx}v + u \frac{d}{dx} = (uv)'$$

$$\int \left( \frac{d}{dx}v + u \frac{d}{dx} \right) dx = \int (uv)' dx$$

$$\int v du + \int u dv = uv$$

Hopefully  
 $\int v du$  is  
simpler

$$\int u dv = uv - \int v du$$

You find  $u$  &  $dv$

easy to  
differentiate

easy to  
integrate

$$\int x \sin x dx$$

$$\begin{aligned} u &= x & dv &= \sin x dx \\ du &= dx & v &= -\cos x \end{aligned}$$

#5 1 & 7 are  
like this

$$uv - \int v du = -x \cos x + \int +\cos x dx$$

$$= -x \cos x + \sin x + C$$

$\arctan(v), \ln x = u$ , 90% of the time

$$\int \sin^4 x \, dx$$

$$u = \sin^4 x \quad dv = dx$$

$$du = 4 \sin^3 x \cos x \, dx \quad v = x$$

$$= x \sin^4 x - \int 4x \sin^3 x \cos x \, dx$$

$$u = x$$

$$du = dx$$

Diff

Integrate

$$dv = \sin^3 x \cos x \, dx$$

$$v = \frac{1}{4} \sin^4 x$$

$$= x \sin^4 x - 4 \int \frac{1}{4} \sin^4 x \, dx$$

$$\int \sin^4 x \, dx = x \sin^4 x - \int \sin^4 x \, dx$$



$$2 \int \sin^4 x \, dx = x \sin^4 x$$



$$\int \sin^4 x = \frac{1}{2} x \sin^4 x + C$$

Are these the same?!

$$\int \sin^4 x \, dx$$

$$\sin^4 x = (\sin^2 x)^2$$

$$= \left( \frac{1 - \cos(2x)}{2} \right)^2$$

$$= \frac{1}{4} (1 - \cos(2x))^2$$

$$= \frac{1}{4} (1 - 2\cos(2x) + \cos^2(2x))$$

$$= \frac{1}{4} - \frac{1}{2} \cos(2x) + \frac{1}{4} \cos^2(2x)$$

$$= \frac{1}{4} - \frac{1}{2} \cos(2x) + \frac{1}{8} (1 + \cos(4x))$$

$$= \frac{1}{4} - \frac{1}{2} \cos(2x) + \frac{1}{8} + \frac{1}{8} \cos(4x)$$

$$= \frac{3}{8} - \frac{1}{2} \cos(2x) + \frac{1}{8} \cos(4x)$$

$$\int \left( \frac{3}{8} - \frac{1}{2} \cos(2x) + \frac{1}{8} \cos(4x) \right) dx$$

$$= \frac{3}{8} x - \frac{1}{4} \sin(2x) + \frac{1}{32} \cos(4x) + C$$

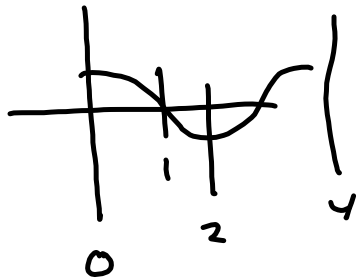
Come grab notes & videos, direct,  
in my office, to save time (downloads)  
on the Internet.

Finish 7.1 by tomorrow  
Tomorrow Nite, begin 7.2.

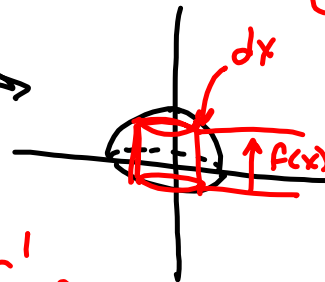
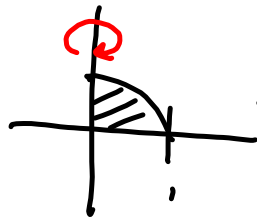
#61

$$y = \cos \frac{\pi x}{2} \quad y = 0, \quad 0 \leq x \leq 1,$$

Revolve about y-axis



$$\frac{2\pi}{\pi} = 4$$



Cylindrical Shells

$$V = 2\pi \int_0^1 x f(x) dx$$

$$u = x \quad du = dx$$

$$dv = \cos \frac{\pi x}{2} dx$$

$$v = \frac{2}{\pi} \sin \frac{\pi x}{2} = 2\pi \int_0^1 x \cos \left( \frac{\pi x}{2} \right) dx$$

$$= 2\pi \left[ uv - \int v du \right]$$

$$= 2\pi \left[ \frac{2x}{\pi} \sin \frac{\pi x}{2} - \int \frac{2}{\pi} \sin \left( \frac{\pi x}{2} \right) dx \right]$$

$$= 2\pi \left( \frac{2x}{\pi} \sin \frac{\pi x}{2} \right) - 2\pi \cdot \frac{2}{\pi} \cdot \frac{2}{\pi} \int \sin \frac{\pi x}{2} \cdot \frac{\pi}{2} dx$$

$$= 4x \sin \frac{\pi x}{2} - \frac{8}{\pi} \left( -\cos \frac{\pi x}{2} \right) + C$$

$$= 4x \sin \left( \frac{\pi x}{2} \right) + \frac{8}{\pi} \cos \left( \frac{\pi x}{2} \right) + C$$