§7.1 Theory & Exercises thru #47
are posted in video & notes.

I'm interested in #47, today
u, v are functions of x. Then

$(uv)' = u'v + uv'$

$\frac{d}{dx}[uv] = \frac{du}{dx}v + u\frac{dv}{dx} = (uv)'$

$\int\left(\frac{du}{dx}v + u\frac{dv}{dx}\right)dx = \int (uv)'\,dx$

$\int v\,du + \int u\,dv = uv$

Hopefully $\int v\,du$ is simpler

$\int u\,dv = uv - \int v\,du$

You find u & dv

u: easy to differentiate

dv: easy to integrate

$\int x \sin x\,dx$

#s 1 & 7 are like this

$u = x$ $dv = \sin x\,dx$

$du = dx$ $v = -\cos x$

$uv - \int v\,du = -x\cos x + \int +\cos x\,dx$

$= -x\cos x + \sin x + C$

arctan(x), ln x = u, 90% of the time

$$\int \sin^4 x\, dx$$

$u = \sin^4 x \qquad dv = dx$

$du = 4\sin^3 x \cos x\, dx \qquad v = x$

$$= x\sin^4 x - \int 4x \boxed{\sin^3 x \cos x\, dx}$$

Diff: $u = x$, $du = dx$

Integrate: $dv = \sin^3 x \cos x\, dx$, $v = \frac{1}{4}\sin^4 x$

$$= x\sin^4 x - 4\int \frac{1}{4}\sin^4 x\, dx$$

$$\int \sin^4 x\, dx = x\sin^4 x - \int \sin^4 x\, dx$$

$$\Rightarrow 2\int \sin^4 x\, dx = x\sin^4 x$$

$$\Rightarrow \int \sin^4 x = \frac{1}{2}x\sin^4 x + C$$

Are these the same?!

$$\int \sin^4 x\, dx$$

$$\sin^4 x = (\sin^2 x)^2$$

$$= \left(\frac{1-\cos(2x)}{2}\right)^2$$

$$= \frac{1}{4}(1-\cos(2x))^2$$

$$= \frac{1}{4}(1 - 2\cos(2x) + \cos^2(2x))$$

$$= \frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{4}\cos^2(2x)$$

$$= \frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{8}(1+\cos(4x))$$

$$= \frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{8} + \frac{1}{8}\cos(4x)$$

$$= \frac{3}{8} - \frac{1}{2}\cos(2x) + \frac{1}{8}\cos(4x)$$

$$\int \left(3/8 - \frac{1}{2}\cos(2x) + \frac{1}{8}\cos(4x)\right)dx$$

$$= \boxed{\frac{3}{8}x - \frac{1}{4}\sin(2x) + \frac{1}{32}\cos(4x) + C}$$