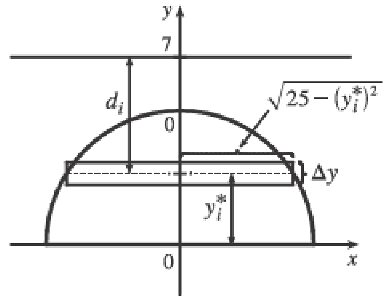
9. Set up coordinate axes as shown in the figure. The length of the ith strip is $2\sqrt{25-(y_i^*)^2}$ and its area is $2\sqrt{25-(y_i^*)^2}\,\Delta y$. The pressure on this strip is approximately $\delta d_i = 62.5(7-y_i^*)$ and so the force on the strip is approximately $62.5(7-y_i^*)2\sqrt{25-(y_i^*)^2}\,\Delta y$. The total force

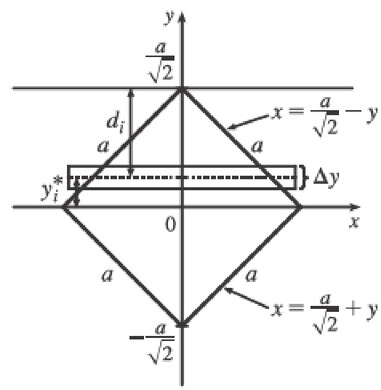


$$F=\lim_{n\to\infty}\sum_{i=1}^{n}62.5(7-y_i^*)2\sqrt{25-(y_i^*)^2}\,\Delta y=125\int_0^5(7-y)\sqrt{25-y^2}\,dy$$

$$=125\left\{\int_0^5 7\sqrt{25-y^2}\,dy-\int_0^5 y\sqrt{25-y^2}\,dy\right\}=125\left\{7\int_0^5\sqrt{25-y^2}\,dy-\left[-\tfrac{1}{3}(25-y^2)^{3/2}\right]_0^5\right\}$$

$$=125\left\{7\left(\tfrac{1}{4}\pi\cdot 5^2\right)+\tfrac{1}{3}(0-125)\right\}=125\left(\tfrac{175\pi}{4}-\tfrac{125}{3}\right)\approx 11{,}972\approx 1.2\times 10^4\text{ lb}$$

10. Set up coordinate axes as shown in the figure. For the *top half*, the length of the ith strip is $2\left(a/\sqrt{2}-y_i^*\right)$ and its area is $2\left(a/\sqrt{2}-y_i^*\right)\Delta y$. The pressure on this strip is approximately $\delta d_i=\delta\left(a/\sqrt{2}-y_i^*\right)$ and so the force on the strip is approximately $2\delta\left(a/\sqrt{2}-y_i^*\right)^2\Delta y$. The total force



$$F_1=\lim_{n\to\infty}\sum_{i=1}^{n}2\delta\left(\frac{a}{\sqrt{2}}-y_i^*\right)^2\Delta y=2\delta\int_0^{a/\sqrt{2}}\left(\frac{a}{\sqrt{2}}-y\right)^2dy$$

$$=2\delta\left[-\frac{1}{3}\left(\frac{a}{\sqrt{2}}-y\right)^3\right]_0^{a/\sqrt{2}}=-\frac{2}{3}\delta\left[0-\left(\frac{a}{\sqrt{2}}\right)^3\right]=\frac{2\delta}{3}\frac{a^3}{2\sqrt{2}}=\frac{\sqrt{2}\,a^3\delta}{6}$$

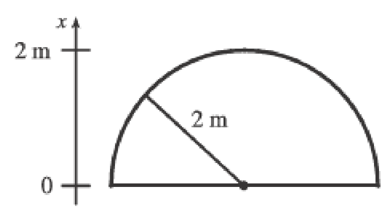
For the *bottom half*, the length is $2\left(a/\sqrt{2}+y_i^*\right)$ and the total force is

$$F_2=\lim_{n\to\infty}\sum_{i=1}^{n}2\delta\left(\frac{a}{\sqrt{2}}+y_i^*\right)\left(\frac{a}{\sqrt{2}}-y_i^*\right)\Delta y=2\delta\int_{-a/\sqrt{2}}^{0}\left(\frac{a^2}{2}-y^2\right)dy=2\delta\left[\tfrac{1}{2}a^2y-\tfrac{1}{3}y^3\right]_{-a/\sqrt{2}}^{0}$$

$$=2\delta\left[0-\left(-\frac{\sqrt{2}\,a^3}{4}+\frac{\sqrt{2}\,a^3}{12}\right)\right]=2\delta\left(\frac{\sqrt{2}\,a^3}{6}\right)=\frac{2\sqrt{2}\,a^3\delta}{6}\qquad [F_2=2F_1]$$

Thus, the total force $F=F_1+F_2=\dfrac{3\sqrt{2}\,a^3\delta}{6}=\dfrac{\sqrt{2}\,a^3\delta}{2}$.

14. $F=\int_0^2\rho g(10-x)2\sqrt{4-x^2}\,dx$



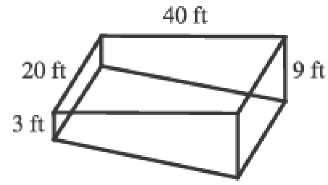
$$=20\rho g\int_0^2\sqrt{4-x^2}\,dx-\rho g\int_0^2\sqrt{4-x^2}\,2x\,dx$$

$$=20\rho g\tfrac{1}{4}\pi(2^2)-\rho g\int_0^4 u^{1/2}\,du\qquad [u=4-x^2,\ du=-2x\,dx]$$

$$=20\pi\rho g-\tfrac{2}{3}\rho g\left[u^{3/2}\right]_0^4=20\pi\rho g-\tfrac{16}{3}\rho g=\rho g\left(20\pi-\tfrac{16}{3}\right)$$

$$=(1000)(9.8)\left(20\pi-\tfrac{16}{3}\right)\approx 5.63\times 10^5\text{ N}$$

17. (a) The area of a strip is $20\,\Delta x$ and the pressure on it is δx_i.



$$F = \int_0^3 \delta x 20\,dx = 20\delta\left[\tfrac{1}{2}x^2\right]_0^3 = 20\delta \cdot \tfrac{9}{2} = 90\delta$$
$$= 90(62.5) = 5625 \text{ lb} \approx 5.63 \times 10^3 \text{ lb}$$

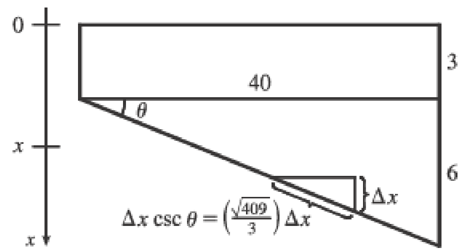
(b) $F = \int_0^9 \delta x 20\,dx = 20\delta\left[\tfrac{1}{2}x^2\right]_0^9 = 20\delta \cdot \tfrac{81}{2} = 810\delta = 810(62.5) = 50{,}625 \text{ lb} \approx 5.06 \times 10^4 \text{ lb}.$

(c) For the first 3 ft, the length of the side is constant at 40 ft. For $3 < x \le 9$, we can use similar triangles to find the length a:

$$\frac{a}{40} = \frac{9-x}{6} \quad \Rightarrow \quad a = 40 \cdot \frac{9-x}{6}.$$

$$F = \int_0^3 \delta x 40\,dx + \int_3^9 \delta x(40)\,\frac{9-x}{6}\,dx = 40\delta\left[\tfrac{1}{2}x^2\right]_0^3 + \tfrac{20}{3}\delta\int_3^9 (9x - x^2)\,dx = 180\delta + \tfrac{20}{3}\delta\left[\tfrac{9}{2}x^2 - \tfrac{1}{3}x^3\right]_3^9$$
$$= 180\delta + \tfrac{20}{3}\delta\left[\left(\tfrac{729}{2} - 243\right) - \left(\tfrac{81}{2} - 9\right)\right] = 180\delta + 600\delta = 780\delta = 780(62.5) = 48{,}750 \text{ lb} \approx 4.88 \times 10^4 \text{ lb}$$

(d) For any right triangle with hypotenuse on the bottom,



$$\sin\theta = \frac{\Delta x}{\text{hypotenuse}} \quad \Rightarrow$$

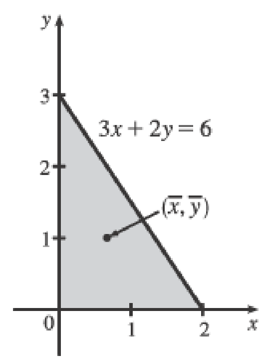
$$\text{hypotenuse} = \Delta x \csc\theta = \Delta x\,\frac{\sqrt{40^2 + 6^2}}{6} = \frac{\sqrt{409}}{3}\,\Delta x.$$

$$F = \int_3^9 \delta x 20\,\tfrac{\sqrt{409}}{3}\,dx = \tfrac{1}{3}\left(20\sqrt{409}\right)\delta\left[\tfrac{1}{2}x^2\right]_3^9$$
$$= \tfrac{1}{3} \cdot 10\sqrt{409}\,\delta(81 - 9) \approx 303{,}356 \text{ lb} \approx 3.03 \times 10^5 \text{ lb}$$

24. $M_x = \sum_{i=1}^{4} m_i y_i = 6(-2) + 5(4) + 1(-7) + 4(-1) = -3$, $M_y = \sum_{i=1}^{4} m_i x_i = 6(1) + 5(3) + 1(-3) + 4(6) = 42$,

and $m = \sum_{i=1}^{4} m_i = 16$, so $\overline{x} = \dfrac{M_y}{m} = \dfrac{42}{16} = \dfrac{21}{8}$ and $\overline{y} = \dfrac{M_x}{m} = -\dfrac{3}{16}$; the center of mass is $(\overline{x}, \overline{y}) = \left(\frac{21}{8}, -\frac{3}{16}\right)$.

26. The region in the figure is "left-heavy" and "bottom-heavy," so we know $\overline{x} < 1$ and $\overline{y} < 1.5$, and we might guess that $\overline{x} = 0.7$ and $\overline{y} = 1.2$.



$$3x + 2y = 6 \quad \Leftrightarrow \quad 2y = 6 - 3x \quad \Leftrightarrow \quad y = 3 - \tfrac{3}{2}x.$$

$$A = \int_0^2 \left(3 - \tfrac{3}{2}x\right)dx = \left[3x - \tfrac{3}{4}x^2\right]_0^2 = 6 - 3 = 3.$$

$$\overline{x} = \tfrac{1}{A}\int_0^2 x\left(3 - \tfrac{3}{2}x\right)dx = \tfrac{1}{3}\int_0^2 \left(3x - \tfrac{3}{2}x^2\right)dx = \tfrac{1}{3}\left[\tfrac{3}{2}x^2 - \tfrac{1}{2}x^3\right]_0^2$$
$$= \tfrac{1}{3}(6 - 4) = \tfrac{2}{3}.$$

$$\overline{y} = \tfrac{1}{A}\int_0^2 \tfrac{1}{2}\left(3 - \tfrac{3}{2}x\right)^2 dx = \tfrac{1}{3} \cdot \tfrac{1}{2}\int_0^2 \left(9 - 9x + \tfrac{9}{4}x^2\right)dx = \tfrac{1}{6}\left[9x - \tfrac{9}{2}x^2 + \tfrac{3}{4}x^3\right]_0^2 = \tfrac{1}{6}(18 - 18 + 6) = 1.$$

Thus, the centroid is $(\overline{x}, \overline{y}) = \left(\tfrac{2}{3}, 1\right)$.