**1.** $y = x^4 \quad \Rightarrow \quad dy/dx = 4x^3 \quad \Rightarrow \quad ds = \sqrt{1 + (dy/dx)^2}\,dx = \sqrt{1 + 16x^6}\,dx$

(a) By (7), an integral for the area of the surface obtained by rotating the curve about the $x$-axis is

$S = \int 2\pi y\,ds = \int_0^1 2\pi x^4 \sqrt{1 + 16x^6}\,dx.$

(b) By (8), an integral for the area of the surface obtained by rotating the curve about the $y$-axis is

$S = \int 2\pi x\,ds = \int_0^1 2\pi x \sqrt{1 + 16x^6}\,dx.$

**4.** $x = \sqrt{y - y^2}$ [defined for $0 \le y \le 1$] $\Rightarrow$

$$\frac{dx}{dy} = \frac{1 - 2y}{2\sqrt{y - y^2}} \quad \Rightarrow \quad ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy = \sqrt{\frac{4(y - y^2) + 1 - 4y + 4y^2}{4(y - y^2)}}\,dy = \sqrt{\frac{1}{4y(1 - y)}}\,dy.$$

(a) By (7), $S = \int 2\pi y\,ds = \int_0^1 2\pi y \sqrt{\dfrac{1}{4y(1 - y)}}\,dy.$

(b) By (8), $S = \int 2\pi x\,ds = \int_0^1 2\pi \sqrt{y - y^2} \sqrt{\dfrac{1}{4y(1 - y)}}\,dy.$

**5.** $y = x^3 \quad \Rightarrow \quad y' = 3x^2$. So

$$S = \int_0^2 2\pi y \sqrt{1 + (y')^2}\,dx = 2\pi \int_0^2 x^3 \sqrt{1 + 9x^4}\,dx \qquad [u = 1 + 9x^4,\ du = 36x^3\,dx]$$

$$= \tfrac{2\pi}{36} \int_1^{145} \sqrt{u}\,du = \tfrac{\pi}{18} \left[\tfrac{2}{3} u^{3/2}\right]_1^{145} = \tfrac{\pi}{27}\left(145\sqrt{145} - 1\right)$$

**12.** $x = 1 + 2y^2 \quad \Rightarrow \quad 1 + (dx/dy)^2 = 1 + (4y)^2 = 1 + 16y^2.$

So $S = 2\pi \int_1^2 y \sqrt{1 + 16y^2}\,dy = \frac{\pi}{16} \int_1^2 (16y^2 + 1)^{1/2} 32y\,dy = \frac{\pi}{16}\left[\frac{2}{3}(16y^2 + 1)^{3/2}\right]_1^2 = \frac{\pi}{24}\left(65\sqrt{65} - 17\sqrt{17}\right).$

**13.** $y = \sqrt[3]{x} \quad \Rightarrow \quad x = y^3 \quad \Rightarrow \quad 1 + (dx/dy)^2 = 1 + 9y^4$. So

$$S = 2\pi \int_1^2 x \sqrt{1 + (dx/dy)^2}\,dy = 2\pi \int_1^2 y^3 \sqrt{1 + 9y^4}\,dy = \tfrac{2\pi}{36} \int_1^2 \sqrt{1 + 9y^4}\,36y^3\,dy = \tfrac{\pi}{18}\left[\tfrac{2}{3}\left(1 + 9y^4\right)^{3/2}\right]_1^2$$

$$= \tfrac{\pi}{27}\left(145\sqrt{145} - 10\sqrt{10}\right)$$

**18.** $y = x + \sqrt{x} \quad \Rightarrow \quad dy/dx = 1 + \frac{1}{2}x^{-1/2} \quad \Rightarrow \quad 1 + (dy/dx)^2 = 2 + x^{-1/2} + \frac{1}{4}x^{-1} \quad \Rightarrow$

$S = \displaystyle\int_1^2 2\pi\left(x + \sqrt{x}\right)\sqrt{2 + \frac{1}{\sqrt{x}} + \frac{1}{4x}}\,dx$. Let $f(x) = \left(x + \sqrt{x}\right)\sqrt{2 + \dfrac{1}{\sqrt{x}} + \dfrac{1}{4x}}$. Since $n = 10$, $\Delta x = \frac{2 - 1}{10} = \frac{1}{10}$.

Then $S \approx S_{10} = 2\pi \cdot \frac{1/10}{3}\left[f(1) + 4f(1.1) + 2f(1.2) + \cdots + 2f(1.8) + 4f(1.9) + f(2)\right] \approx 29.506566.$

The value of the integral produced by a calculator is $29.506568$ (to six decimal places).

**25.** $S = 2\pi \int_1^\infty y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = 2\pi \int_1^\infty \frac{1}{x}\sqrt{1+\frac{1}{x^4}}\,dx = 2\pi \int_1^\infty \frac{\sqrt{x^4+1}}{x^3}\,dx$. Rather than trying to evaluate this integral, note that $\sqrt{x^4+1} > \sqrt{x^4} = x^2$ for $x > 0$. Thus, if the area is finite, $S = 2\pi \int_1^\infty \frac{\sqrt{x^4+1}}{x^3}\,dx > 2\pi \int_1^\infty \frac{x^2}{x^3}\,dx = 2\pi \int_1^\infty \frac{1}{x}\,dx$. But we know that this integral diverges, so the area $S$ is infinite.