2. Using the arc length formula with $y = \sqrt{2 - x^2} \quad \Rightarrow \quad \dfrac{dy}{dx} = -\dfrac{x}{\sqrt{2 - x^2}}$, we get

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \int_0^1 \sqrt{1 + \frac{x^2}{2 - x^2}}\,dx = \int_0^1 \frac{\sqrt{2}\,dx}{\sqrt{2 - x^2}} = \sqrt{2}\int_0^1 \frac{dx}{\sqrt{\left(\sqrt{2}\right)^2 - x^2}}$$

$$= \sqrt{2}\left[\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]_0^1 = \sqrt{2}\left[\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1} 0\right] = \sqrt{2}\left[\tfrac{\pi}{4} - 0\right] = \sqrt{2}\,\tfrac{\pi}{4}$$

The curve is a one-eighth of a circle with radius $\sqrt{2}$, so the length of the arc is $\frac{1}{8}\left(2\pi \cdot \sqrt{2}\right) = \sqrt{2}\,\frac{\pi}{4}$, as above.

4. $y = xe^{-x^2} \quad \Rightarrow \quad dy/dx = xe^{-x^2}(-2x) + e^{-x^2} \cdot 1 = e^{-x^2}(1 - 2x^2) \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + (1 - 2x^2)^2 e^{-2x^2}$.

So $L = \int_0^1 \sqrt{1 + (1 - 2x^2)^2\, e^{-2x^2}}\,dx$.

7. $y = 1 + 6x^{3/2} \quad \Rightarrow \quad dy/dx = 9x^{1/2} \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + 81x$. So

$L = \int_0^1 \sqrt{1 + 81x}\,dx = \int_1^{82} u^{1/2}\left(\frac{1}{81}\,du\right) \quad \begin{bmatrix} u = 1 + 81x, \\ du = 81\,dx \end{bmatrix} \quad = \frac{1}{81} \cdot \frac{2}{3}\left[u^{3/2}\right]_1^{82} = \frac{2}{243}\left(82\sqrt{82} - 1\right)$

10. $x = \dfrac{y^4}{8} + \dfrac{1}{4y^2} \quad \Rightarrow \quad \dfrac{dx}{dy} = \frac{1}{2}y^3 - \frac{1}{2}y^{-3} \quad \Rightarrow$

$1 + (dx/dy)^2 = 1 + \frac{1}{4}y^6 - \frac{1}{2} + \frac{1}{4}y^{-6} = \frac{1}{4}y^6 + \frac{1}{2} + \frac{1}{4}y^{-6} = \left(\frac{1}{2}y^3 + \frac{1}{2}y^{-3}\right)^2$. So

$L = \int_1^2 \sqrt{\left(\frac{1}{2}y^3 + \frac{1}{2}y^{-3}\right)^2}\,dy = \int_1^2 \left(\frac{1}{2}y^3 + \frac{1}{2}y^{-3}\right)dy = \left[\frac{1}{8}y^4 - \frac{1}{4}y^{-2}\right]_1^2 = \left(2 - \frac{1}{16}\right) - \left(\frac{1}{8} - \frac{1}{4}\right)$
$= 2 + \frac{1}{16} = \frac{33}{16}$.

13. $y = \ln(\sec x) \quad \Rightarrow \quad \dfrac{dy}{dx} = \dfrac{\sec x \tan x}{\sec x} = \tan x \quad \Rightarrow \quad 1 + \left(\dfrac{dy}{dx}\right)^2 = 1 + \tan^2 x = \sec^2 x$, so

$L = \int_0^{\pi/4} \sqrt{\sec^2 x}\,dx = \int_0^{\pi/4} |\sec x|\,dx = \int_0^{\pi/4} \sec x\,dx = \Big[\ln(\sec x + \tan x)\Big]_0^{\pi/4}$
$= \ln\left(\sqrt{2} + 1\right) - \ln(1 + 0) = \ln\left(\sqrt{2} + 1\right)$

19. $y = \frac{1}{2}x^2 \quad \Rightarrow \quad dy/dx = x \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + x^2$. So

$L = \int_{-1}^1 \sqrt{1 + x^2}\,dx = 2\int_0^1 \sqrt{1 + x^2}\,dx$ [by symmetry] $\overset{21}{=} 2\left[\frac{x}{2}\sqrt{1 + x^2} + \frac{1}{2}\ln\left(x + \sqrt{1 + x^2}\right)\right]_0^1 \quad \begin{bmatrix} \text{or substitute} \\ x = \tan\theta \end{bmatrix}$
$= 2\left[\left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\ln\left(1 + \sqrt{2}\right)\right) - \left(0 + \frac{1}{2}\ln 1\right)\right] = \sqrt{2} + \ln\left(1 + \sqrt{2}\right)$

25. $y = \sec x \Rightarrow dy/dx = \sec x \tan x \Rightarrow L = \int_0^{\pi/3} f(x)\,dx$, where $f(x) = \sqrt{1 + \sec^2 x \tan^2 x}$.

Since $n = 10$, $\Delta x = \dfrac{\pi/3 - 0}{10} = \dfrac{\pi}{30}$. Now

$$L \approx S_{10} = \frac{\pi/30}{3}\left[f(0) + 4f\left(\frac{\pi}{30}\right) + 2f\left(\frac{2\pi}{30}\right) + 4f\left(\frac{3\pi}{30}\right) + 2f\left(\frac{4\pi}{30}\right) + 4f\left(\frac{5\pi}{30}\right)\right.$$
$$\left. + 2f\left(\frac{6\pi}{30}\right) + 4f\left(\frac{7\pi}{30}\right) + 2f\left(\frac{8\pi}{30}\right) + 4f\left(\frac{9\pi}{30}\right) + f\left(\frac{\pi}{3}\right)\right] \approx 1.569619.$$

The value of the integral produced by a calculator is 1.569259 (to six decimal places).

40. (a) $y = c + a\cosh\left(\frac{x}{a}\right) \Rightarrow y' = \sinh\left(\frac{x}{a}\right) \Rightarrow 1 + (y')^2 = 1 + \sinh^2\left(\frac{x}{a}\right) = \cosh^2\left(\frac{x}{a}\right)$. So

$L = \int_{-b}^{b} \sqrt{\cosh^2\left(\frac{x}{a}\right)}\,dx = 2\int_0^b \cosh\left(\frac{x}{a}\right)dx = 2\left[a\sinh\left(\frac{x}{a}\right)\right]_0^b = 2a\sinh\left(\frac{b}{a}\right)$.

(b) At $x = 0$, $y = c + a$, so $c + a = 20$. The poles are 50 ft apart, so $b = 25$, and $L = 51 \Rightarrow 51 = 2a\sinh(b/a)$ [from part (a)]. From the figure, we see that $y = 51$ intersects $y = 2x\sinh(25/x)$ at $x \approx 72.3843$ for $x > 0$. So $a \approx 72.3843$ and the wire should be attached at a distance of $y = c + a\cosh(25/a) = 20 - a + a\cosh(25/a) \approx 24.36$ ft above the ground.