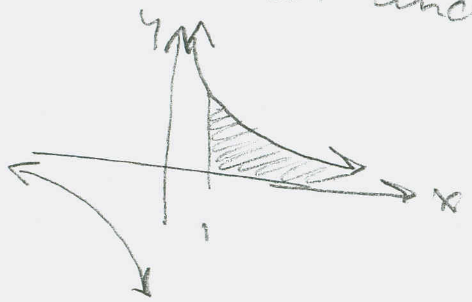
202 §8.8 #s 3, 5, 8, 11, 14, 18, 21, 25, 28, 36

(3) Find the area under $y = \frac{1}{x^3}$ from $x=1$ to $x=t$ and evaluate for $t = 10, 100, 1000$. Then find the total area under the curve for $t \geq 1$.



$$\int_1^t x^{-3}dx = -\frac{1}{2}x^{-2}\Big]_1^t$$

$$= -\frac{1}{2}\left[\frac{1}{t^2} - \frac{1}{1^2}\right] = -\frac{1}{2t^2} + \frac{1}{2}$$

$$= \boxed{\frac{1}{2} - \frac{1}{2t^2}}$$

$t = 10$: $\frac{1}{2} - \frac{1}{200} = \frac{100-1}{200} = \boxed{\frac{99}{200}}$

$t = 100$: $\frac{1}{2} - \frac{1}{20000} = \frac{10000-1}{20000} = \boxed{\frac{9999}{20000}}$

$t = 1000$: $\frac{1}{2} - \frac{1}{2{,}000{,}000} = \frac{1{,}000{,}000 - 1}{2{,}000{,}000} = \boxed{\frac{999{,}999}{2{,}000{,}000}}$

$t \longrightarrow \infty$: $\boxed{\frac{1}{2}}$

#s 5-40 Is it convergent? If it is, evaluate it.

(5) $\int_1^\infty \frac{dx}{(3x+1)^2}$

Yes. The 2^{nd} power downstairs makes it OK on the infinite interval.

$$= \frac{1}{3}\int_1^\infty \frac{3dx}{(3x+1)^2} =$$

$u = 3x+1 \quad du = 3dx$

$$= \lim_{t\to\infty} \frac{1}{3}\left[-(3x+1)^{-1}\right]_1^t = \frac{1}{3}\lim_{t\to\infty}\left[-\frac{1}{3t+1} - \left(-\frac{1}{3(1)+1}\right)\right]$$

$$= \frac{1}{3}\cdot\frac{1}{4} = \boxed{\frac{1}{12}}$$

202 § 8.8 #s 11, 14, 18, 21, 25, 28, 36

(11) $\int_{-\infty}^{\infty} \frac{x}{1+x^2}\,dx$ I'm thinking NOT, because it's like a $\frac{1}{x}$ as $x \to \pm\infty$ and we need the power of $x > 1$ in the denominator. But let's see.

$= \lim_{t\to\infty} \frac{1}{2}\int_{-t}^{t} \frac{2x\,dx}{x^2+1}$

$= \frac{1}{2}\lim_{t\to\infty} \left[\ln(x^2+1)\right]_{-t}^{t} = \frac{1}{2}\lim_{t\to\infty}\left[\ln(t^2+1) - \ln(t^2+1)\right]$

$= \frac{1}{2}\left[\infty - \infty\right]$!? This one is a bit dicey, since it's an odd function on a symmetric interval, so it should come out <u>zero</u>. But we want to be precise in our handling of it. Text method is to break it into two pieces:

$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx$ and <u>BOTH</u> pieces must converge in order for the sum to converge.

(14) $\int_1^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = -2\lim_{t\to\infty}\int_1^t e^{-\sqrt{x}}\cdot\left(-\frac{1}{2\sqrt{x}}\,dx\right)$

$= -2\lim_{t\to\infty}\left[e^{-\sqrt{x}}\right]_1^t = -2\lim_{t\to\infty}\left[e^{-\sqrt{t}} - e^{-1}\right] = \boxed{\frac{2}{e}}$

202 §8.8 #s 18, 21, 25, 28, 36

(18) $\displaystyle\int_0^\infty \frac{dz}{z^2+3z+2}$

Should converge on ∞ interval, b/c the "net power" is -2.

$$\frac{1}{(z+1)(z+2)} = \frac{A}{z+1} + \frac{B}{z+2}$$

$1 = Az + 2A + Bz + B$

$A + B = 0 \Longrightarrow A = -B$

$2A + B = 1$

$2A - A = 1$

$$\boxed{\begin{matrix} A = 1 \\ B = -1 \end{matrix}}$$

This gives

$$\lim_{t\to\infty} \int_0^t \left(\frac{1}{z+1} - \frac{1}{z+2}\right) dz$$

$$= \lim_{t\to\infty} \Big[\ln(z+1) - \ln(z+2)\Big]_0^t$$

$$= \lim_{t\to\infty} \Big[\ln(t+1) - \ln(t+2) - \big(\ln(1) - \ln(2)\big)\Big]$$

$$= \lim_{t\to\infty} \left[\ln\left(\frac{t+1}{t+2}\right) + \ln(2)\right]$$

$$= \ln(1) + \ln(2) = \boxed{\ln(2)}$$

(21) $\displaystyle\int_1^\infty \frac{\ln x}{x}\, dx = \int_1 \ln(x) \cdot \frac{dx}{x}$

$u = \ln x \longrightarrow du = \frac{dx}{x}$

$x = 1 \longrightarrow u = \ln(1) = 0$

"$x = \infty \longrightarrow u = \ln(\infty) = \infty$" !?

$$= \int_0^\infty u\, du \;\cancel{\Longrightarrow}\; \text{Divergent}$$

RECOGNITION: ~~The~~ $\ln(x)$ grows without bound and $\int \frac{1}{x}\, dx$ is Divergent.

So, we reckon it diverges

202 §8.8 #s 25, 28, 36

(25) $\displaystyle\int_e^\infty \frac{1}{x(\ln(x))^2}\,dx$

Should converge. $x(\ln(x))^2$ grows faster than x, so $\frac{1}{x(\ln(x))^2}$ shrinks faster than $\frac{1}{x}$

Let $u = \ln(x)$

Then $du = \frac{1}{x}\,dx$

$x = e \Rightarrow u = \ln(e) = 1$

$x \to \infty \Rightarrow u = \ln(x) \to \infty$

This gives

$\displaystyle\int_1^\infty u^{-2}\,du$ which converges by p-test, as we anticipated, above.

$$= \lim_{t\to\infty}\left[-u^{-1}\right]_1^t = \lim_{t\to\infty}\left[-\frac{1}{t} - \left(-\frac{1}{1}\right)\right] = \boxed{1}$$

DRAT! It was $\displaystyle\int_e^\infty \frac{1}{x(\ln(x))^3}\,dx$.

Pretty much the same deal.

$$= -\frac{1}{2}\lim_{t\to\infty}\left[u^{-2}\right]_1^t = -\frac{1}{2}\lim_{t\to\infty}\left[\frac{1}{t^2} - \frac{1}{1^2}\right] = \boxed{+\frac{1}{2}}$$

202 §8.8 #s 28, 36

28 $\displaystyle\int_2^3 \frac{dx}{\sqrt{3-x}}$

$u = 3-x \longrightarrow du = -dx$

$x=2 \longrightarrow 3-x = 3-2 = 1 = u$

$x=3 \longrightarrow 3-x = 3-3 = 0 = u$

$$= -\int_1^0 \frac{du}{\sqrt{u}} = \int_0^1 \frac{du}{\sqrt{u}} = \underbrace{\int_0^1 u^{-\frac{1}{2}}\,du = 2u^{\frac{1}{2}}\Big]_0^1}_{\text{sketchy. Re-write:}} = \boxed{2}$$

$$= \lim_{t\to 0^+} \int_t^1 u^{-\frac{1}{2}}\,du = \lim_{t\to 0^+}\left[2u^{\frac{1}{2}}\right]_t^1 = 2$$

we need the limit in there, because the integrand

$\frac{1}{\sqrt{u}}$ is NOT DEFINED at $u=0$.

36 $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \csc x\,dx$

$\csc x$ is not defined @ $x=\pi$

FORGOT $\displaystyle\int \csc x\,dx = \int \frac{\csc x\cot x + \csc^2 x}{\cot x + \csc x}\,dx$

$\displaystyle = -\int \frac{-\csc x\cot x - \csc^2 x}{\cot x + \csc x}\,dx$ OK

$= -\ln|\cot x + \csc x| + C$

$= -\ln\left|\frac{\cos x + 1}{\sin x}\right| + C = \ln\left|\frac{\sin x}{\cos x + 1}\right|$

$$= \lim_{t\to\pi} \int_{\frac{\pi}{2}}^{t} \csc x\,dx$$

$$= -\lim_{t\to\infty}\Big[\ln|\csc x + \cot x|\Big]_{\frac{\pi}{2}}^{t}$$

$$= -\lim_{t\to\infty}\left[\ln\left|\frac{1+\cos x}{\sin x}\right|\right]_{\frac{\pi}{2}}^{t}$$

Diverges