202 §8.7 #s 7, 12, 19, 20, 22, 30

#s 7–18 Use the (a) Trapezoidal Rule (b) Midpoint Rule and (c) Simpson's Rule to approximate the integral with the given value of n. To 6 places.

(7) $\int_0^2 \sqrt[4]{x^2+1}\, dx = I \quad n = 8 \qquad \frac{b-a}{n} = \frac{2}{8} = \frac{1}{4} = \Delta x$

$x_0 = 0 \quad f(x_0) = \sqrt[4]{1} = 1$

$x_1 = \frac{1}{4} \quad f(\frac{1}{4}) = \sqrt[4]{\frac{17}{16}} = \frac{\sqrt[4]{17}}{2} \approx 1.015271592$

$x_2 = \frac{1}{2} \quad f(\frac{1}{2}) = \sqrt[4]{\frac{5}{4}} \approx 1.057371263$

$x_3 = \frac{3}{4} \quad f(\frac{3}{4}) = \sqrt[4]{\frac{25}{16}} \approx 1.118033989$

$x_4 = 1 \quad f(1) = \sqrt[4]{2} \approx 1.189207115$

$x_5 = \frac{5}{4} \quad f(\frac{5}{4}) = \sqrt[4]{\frac{41}{16}} \approx 1.265219767$

$x_6 = \frac{3}{2} \quad f(\frac{3}{2}) = \sqrt[4]{\frac{13}{4}} \approx 1.342674807$

$x_7 = \frac{7}{4} \quad f(\frac{7}{4}) = \sqrt[4]{\frac{65}{16}} \approx 1.419705757$

$x_8 = 2 \quad f(2) = \sqrt[4]{3} \approx 1.495348781$

$$I \approx \frac{\frac{1}{4}}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_7) + f(x_8)\right]$$

$$\approx \frac{1}{8}\Big[1 + 2(1.015271592) + 2(1.057371263) + 2(1.118033989) + 2(1.189207115) + 2(1.265219767) + 2(1.342674807) + 2(1.419705757) + 1.495348781\Big] \approx \boxed{2.4137896\overline{7}} \approx T_8$$

$\approx \boxed{2.413790} \approx$

$\int^{202}$ 8.7 #s 12, 19, 20, 22, 30

7 cnt'd   DONE WITH Trapezoidal

MIDPOINT:

$\bar{x}_1 = \frac{0+\frac{1}{4}}{2} = \frac{1}{8}$

$\bar{x}_2 = \bar{x}_1 + \Delta x = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

$\bar{x}_3 = \frac{5}{8}$

$\bar{x}_4 = \frac{7}{8}$

$\bar{x}_5 = \frac{9}{8}$

$\bar{x}_6 = \frac{11}{8}$

$\bar{x}_7 = \frac{13}{8}$

$\bar{x}_8 = \frac{15}{8}$

$$\text{Area} \approx \Delta x \left[\sum_{k=1}^{8} f(\bar{x}_k) \right] =$$

$$= \frac{1}{4}\left[f\left(\frac{1}{8}\right) + f\left(\frac{3}{8}\right) + f\left(\frac{5}{8}\right) + \cdots + f\left(\frac{15}{8}\right) \right]$$

$$\approx \frac{1}{4}\left[1.003883568 + \cdots + 1.457737974 \right]$$

$$= \frac{1}{4}\left[9.645812032 \right] \approx \boxed{2.411453008} \approx M_8$$

$$\boxed{\approx 2.411453}$$

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Simpson's for #7. x_k's same as (i)

$$S_8 = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_7) + f(x_8)\right]$$

$$= \frac{\frac{1}{4}}{3}\left[\quad\right] = \frac{1}{12}\left[\quad\right]$$

$$\approx \frac{1}{12}\Big[\underbrace{1}_{f(0)} + 4(\underbrace{1.015271592}_{f(\frac{1}{8})}) + 2(\underbrace{1.057371263}_{f(\frac{1}{4})})$$

$$+ 4(\underbrace{1.118033989}_{f(\frac{3}{8})}) + 2(\underbrace{1.189207115}_{f(\frac{1}{2})})$$

$$+ 4(\underbrace{1.265219767}_{f(\frac{5}{8})}) + 2(\underbrace{1.342674807}_{f(\frac{3}{4})})$$

$$+ 4(\underbrace{1.419705757}_{f(\frac{7}{8})} + \underbrace{1.495348781}_{f(2)}\Big]$$

$$\approx \frac{1}{12}\left[28.94677957\right] \approx \boxed{2.412231631} \approx S_8$$

$$\approx \boxed{2.412232}$$

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12 $\int_0^4 \sqrt{1+\sqrt{x}}\, dx$, $n=8$

(a) $\Delta x = \frac{4}{8} = \frac{1}{2}$ Trap

$x_0 = 0$

$x_1 = \frac{1}{2}$

$x_2 = 1$

$x_3 = \frac{3}{2}$

$x_4 = 2$

$x_5 = \frac{5}{2}$

$x_6 = 3$

$x_7 = \frac{7}{2}$

$x_8 = 2$

$T_8 \approx \text{Area} \approx \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_7) + f(x_8)\right]$

$= \frac{1}{4}\left[f(x_0) + \sum_{k=1}^{7} 2f(x_k) + f(x_8)\right]$

$\approx \frac{1}{4}\left[24.17193945\right]$

$\approx 6.042984862 \approx T_8$

$\approx \boxed{6.042985 \approx T_8}$

(b) Mid

$\bar{x}_1 = \frac{1}{4}$

$\bar{x}_2 = \frac{3}{4}$

$\bar{x}_3 = \frac{5}{4}$

$\bar{x}_4 = \frac{7}{4}$

$\bar{x}_5 = \frac{9}{4}$

$\bar{x}_6 = \frac{11}{4}$

$\bar{x}_7 = \frac{13}{4}$, $\bar{x}_8 = \frac{15}{4}$

$M_8 \approx \Delta x\left[\sum_{k=1}^{8} f(\bar{x}_k)\right]$

$\approx \frac{1}{2}\left[f(\frac{1}{4}) + \cdots + f(\frac{15}{4})\right]$

$\approx \frac{1}{2}\left[12.16955599\right]$

≈ 6.084777995

$\approx \boxed{6.084778 \approx M_8}$

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12 cnt'd

Simpson's

(c) $S_8 \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 4f(x_7) + f(x_8)\right]$

$\approx \frac{1}{6}\left[36.37006972\right] \approx 6.061678286$

$\approx \boxed{6.061678 \approx S_8}$

20 (a) Find T_{10} and M_{10} for $\int_0^1 \cos(x^2)\,dx$

$x_k = \frac{1}{10}k$

T_{10}: $\Delta x = \frac{1}{10}$

$T_{10} \approx \frac{1}{20}\left[f(x_0) + 2\sum_{k=1}^{9} f(x_k) + f(x_{10})\right]$

$\approx \frac{1}{20}\left[16.06248514\right]$

$\approx$