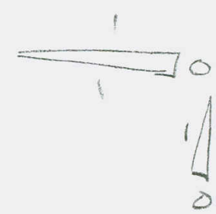
202 §8.6 #s 5, 10, 15, 20, 25, 30, 31

#s 5-30 Use Tables to evaluate the integral

5 $\displaystyle\int_0^1 2x \arccos(x)\,dx = 2\int_0^1 x \arccos(x)\,dx$

$$\overset{\#91}{=} 2\left[\frac{2x^2-1}{4}\arccos(x) - \frac{x\sqrt{1-x^2}}{4}\right]_0^1$$



$$= 2\left[\left(\frac{1}{4}\cdot 0 - \frac{1\sqrt{0}}{4}\right) - \left(-\frac{1}{4}\cdot\frac{\pi}{2} - \frac{0\sqrt{1}}{4}\right)\right]$$

$$= 2\left[0 + \frac{\pi}{8}\right] = \boxed{\frac{\pi}{4}}$$

10 $\displaystyle\int \frac{\sqrt{2y^2-3}}{y^2}\,dy = \int \frac{\sqrt{2}\sqrt{y^2-\frac{3}{2}}}{y^2}\,dy$

$$= \sqrt{2}\int \frac{\sqrt{y^2-\left(\sqrt{\frac{3}{2}}\right)^2}}{y^2}\,dy \overset{\#42}{=} \sqrt{2}\left[-\frac{\sqrt{y^2-\frac{3}{2}}}{y} + \ln\left|y+\sqrt{y^2-\tfrac{3}{2}}\right|\right] + C$$

$$= \sqrt{2}\left[-\frac{\sqrt{y^2-\frac{3}{2}}}{y} + \ln\left|y+\sqrt{y^2-\tfrac{3}{2}}\right|\right] + C$$

$$= -\frac{\sqrt{2y^2-3}}{y} + \sqrt{2}\ln\left|y+\sqrt{y^2-\tfrac{3}{2}}\right| + C$$

Book made the substitution $u = \sqrt{2}\,y$.
I factored $\sqrt{2}$ out of the radical and use $a = \sqrt{\frac{3}{2}}$, instead.

202 §8.6 #s 15, 20, 25, 30, 31

15 $\int e^{2x} \arctan(e^x)\, dx$ Let $u = e^x$. Then $du = e^x dx$

$= \int e^x \arctan(e^x) \cdot e^x dx = \int u \arctan(u)\, du$

$\overset{\#92}{=} \frac{u^2+1}{2}\arctan(u) - \frac{u}{2} + C$

$= \boxed{\frac{e^{2x}+1}{2}\arctan(e^x) - \frac{e^x}{2} + C}$

20 $\int \frac{\sin(2\theta)}{\sqrt{5-\sin\theta}}\, d\theta = \int \frac{2\sin\theta\cos\theta\, d\theta}{\sqrt{5-\sin\theta}}$

$(u = \sin\theta,\ du = \cos\theta\, d\theta)$

$a = 5,\ b = -1$

$= 2\int \frac{u\, du}{\sqrt{5-u}} \overset{\#55}{=} 2\left[\frac{2}{3b^2}(bu-2a)\sqrt{a+bu}\right] + C$

$= 2 \cdot \frac{2}{3}(-u-2(5))\sqrt{5-u} + C$

$= \boxed{\frac{4}{3}(-\sin\theta - 10)\sqrt{5-\sin\theta} + C}$

202 §8.6 #s 25, 30, 31

(25) $\displaystyle\int \frac{\sqrt{4+(\ln(x))^2}}{x}\,dx$ Let $u=\ln x$. Then $du = \dfrac{dx}{x}$

$= \displaystyle\int \sqrt{4+u^2}\,du = \int \sqrt{2^2+u^2}\,du \overset{\#21}{=} \boxed{\frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left(u+\sqrt{a^2+u^2}\right)+C}$

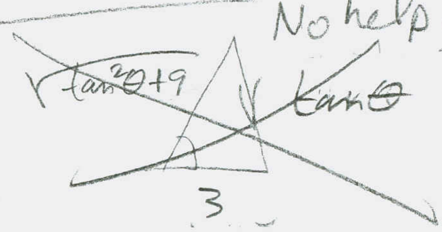
(30) $\displaystyle\int \frac{\sec^2\theta\,\tan^2\theta}{\sqrt{9-\tan^2\theta}}\,d\theta$ Let $u=\tan\theta$ Then $du=\sec^2\theta\,d\theta$

$a=3$

$= \displaystyle\int \frac{u^2\,du}{\sqrt{9-u^2}} \overset{\#34}{=} -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right)+C$

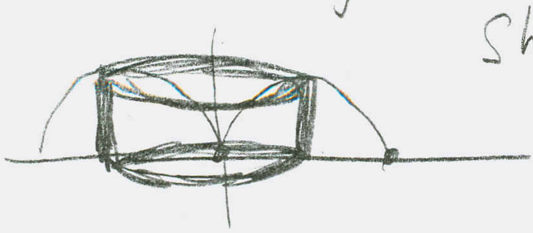
$= \boxed{-\frac{\tan\theta}{2}\sqrt{9-\tan^2\theta} + \frac{9}{2}\arcsin\left(\frac{\tan\theta}{3}\right)+C}$

No help.



(31) Find volume of solid obtained by revolving the region bdd by $y=x\sqrt{4-x^2}$, $x=0$, $x=2$ about the y-axis.

Shell method



$V = l\cdot w\cdot h = 2\pi x \cdot dx \cdot f(x)$

202 §8.6 # 31

$$V = 2\pi \int_a^b x f(x)\,dx = 2\pi \int_0^2 x \cdot x\sqrt{4-x^2}\,dx$$

a = 2

#31

$$= 2\pi \int_0^2 x^2 \sqrt{4-x^2}\,dx = 2\pi \left[\frac{x}{8}(2x^2-4)\sqrt{4-x^2} + \frac{2^4}{8}\arcsin\left(\frac{x}{2}\right)\right]_0^2 =$$

$$2\pi \left[\boxed{\frac{1}{4}x(x^2-2)\sqrt{4-x^2} + 2\arcsin\left(\frac{x}{2}\right)}\right]_0^2$$

oopsie! This is a DEFINITE INTEGRAL!

$$= 2\pi \left[\frac{1}{4}(2)(4-2)\sqrt{4-4} + 2\arcsin(1) - \left(\frac{1}{4}(0) \cdots + 2\arcsin(0)\right)\right] = 2\pi\left[2\,\frac{\pi}{2}\right] = \boxed{2\pi^2}$$