202 §8.5 I #s 2, 7, 12, 17, 22, 27, 32, 37

#s 1-80 Evaluate the integral

(2) $\int \frac{\sin^3 x}{\cos x}\,dx = \int \frac{(1-\cos^2 x)\sin x\,dx}{\cos x} = \int \frac{\sin x\,dx}{\cos x} - \int \sin x \cos x\,dx$

$= -\ln|\cos x| - \frac{\sin^2 x}{2} + C = \ln|\sec x| - \frac{1}{2}\sin^2 x + C$

OR $+\frac{1}{2}\cos^2 x + \ln|\sec x| + \hat{C}$

$\left(-\frac{1}{2}\sin^2 x = -\frac{1}{2}(1-\cos^2 x) = -\frac{1}{2} + \frac{1}{2}\cos^2 x \right.$

so the difference is the $-\frac{1}{2}$, which can be absorbed into the constant $C \rightsquigarrow \hat{C}$.

(7) $\int_{-1}^{1} \frac{e^{\arctan y}}{1+y^2}\,dy$

Let $u = \arctan y$. Then $du = \frac{1}{1+y^2}\,dy$

$y = -1 \Rightarrow$ $u = \arctan(-1) = -\frac{\pi}{4}$

$y = 1 \Rightarrow \arctan(1) = \frac{\pi}{4}$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^u\,du$

$= e^u\Big]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \boxed{e^{\frac{\pi}{4}} - e^{-\frac{\pi}{4}}}$

(12) $\int \frac{x}{x^4+x^2+1}\,dx$ Let $u = x^2 \Rightarrow du = 2x\,dx$

$dx = \frac{du}{2x}$

$= \frac{1}{2}\int \frac{2x\,dx}{u^2+u+1} = \frac{1}{2}\int \frac{du}{u^2+u+1} = \frac{1}{2}\int \frac{du}{(u+\frac{1}{2})^2 - \left(\frac{\sqrt{3}}{2}\right)^2}$

$u^2+u+1 = u^2+u+\left(\frac{1}{2}\right)^2 - \frac{1}{4} + 1 = (u+\frac{1}{2})^2 - \frac{3}{4}$

$= \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\arctan\left(\frac{u+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + C = \boxed{\frac{\sqrt{3}}{3}\arctan\left(\frac{2}{\sqrt{3}}\left(x^2+\frac{1}{2}\right)\right) + C}$

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(17) $\int x \sin^2 x \, dx$

$u = x \quad du = dx$

$dv = \sin^2 x \, dx = \frac{1}{2}(1-\cos(2x))\,dx$

$= uv - \int v\,du$

$\Longrightarrow v = \frac{1}{2}x - \frac{1}{4}\sin(2x)$

$= \frac{1}{2}x - \frac{1}{2}\sin(x)\cos(x)$ may or may not use.

$= x\left(\frac{1}{2}x - \frac{1}{4}\sin(2x)\right) - \int \left(\frac{1}{2}x - \frac{1}{2}\sin(x)\cos(x)\right)dx$

$= \frac{1}{2}x^2 - \frac{1}{4}\sin(2x) - \frac{1}{4}x^2 + \frac{1}{2}\cdot\frac{1}{2}\sin^2(x) + C$

$= \boxed{\frac{1}{4}x^2 - \frac{1}{4}\sin(2x) + \frac{1}{4}\sin^2(x) + C}$ OR $\frac{1}{4}x^2 - \frac{1}{2}\sin(x)\cos x + \frac{1}{4}\sin^2 x + C$

(22) $\int \frac{\ln x}{x\sqrt{1+(\ln x)^2}}\,dx$

Let $u = \ln x \Longrightarrow du = \frac{dx}{x}$

$= \int \frac{u}{\sqrt{1+u^2}}\,du$

Let $v = 1+u^2$. Then $dv = 2u\,du$

$= \frac{1}{2}\int \frac{2u\,du}{\sqrt{v}} = \frac{1}{2}\int \frac{dv}{\sqrt{v}} = \frac{1}{2}\cdot\frac{2}{1}v^{\frac{1}{2}} + C$

$= \sqrt{v} + C = \sqrt{1+u^2} + C = \boxed{\sqrt{1+(\ln(x))^2} + C}$

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27 $\displaystyle\int \frac{dx}{1+e^x}$

Let $u = 1+e^x \Rightarrow du = e^x dx \Rightarrow dx = \frac{du}{e^x}$

$= \frac{du}{u-1}$

$= \displaystyle\int \frac{1}{u} \cdot \frac{du}{u-1}$

$\frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1}$

$1 = Au - A + Bu$

$\boxed{A = -1}$

$A + B = 0$

$-1 + B = 0$

$\boxed{B = 1}$

This gives

$\displaystyle\int \frac{-du}{u} + \int \frac{du}{u-1}$

$= -\ln|u| + \ln|u-1| + C$

$= -\ln|e^x+1| + \ln|e^x+1-1| + C$

$= \boxed{-\ln|e^x+1| + x + C}$

32 $\displaystyle\int \frac{\sqrt{2x-1}}{2x+3}\,dx$

Let $u = 2x-1 \Rightarrow du = 2dx$

$2x+3 = 2x-1+4 = u+4$

Let $v = \sqrt{u} \Rightarrow dv = \frac{1}{2\sqrt{u}}du$

$\Rightarrow du = 2\sqrt{u}\,dv = 2v\,dv$

$v^2 = u \Rightarrow v^2+4 = u+4$

$= \frac{1}{2}\displaystyle\int \frac{\sqrt{u}}{u+4}\,du$

$= \frac{1}{2}\displaystyle\int \frac{v}{v^2+4} \cdot 2v\,dv$

$= \frac{2}{2}\displaystyle\int \frac{v^2}{v^2+4}\,dv$

$$\begin{array}{r|l} & 1 \quad r-4 \\ v^2+4 & v^2+0v+0 \\ & -(v^2 \quad\quad +4) \\ \hline & -4 \end{array}$$

This gives

$\displaystyle\int \left(1 - \frac{4}{v^2+4}\right)dv = v - 4 \cdot \frac{1}{2}\arctan\left(\frac{v}{2}\right) + C$

$= v - 2\arctan\left(\frac{v}{2}\right) + C$

$= \sqrt{u} - 2\arctan\left(\frac{\sqrt{u}}{2}\right) + C$

$= \sqrt{2x-1} - 2\arctan\left(\frac{\sqrt{2x-1}}{2}\right) + C$

202 $\int$ 8.5 I #37

37 $\displaystyle\int_0^{\frac{\pi}{4}} \cos^2\theta \tan^2\theta \, d\theta = \int_0^{\frac{\pi}{4}} \sin^2\theta \, d\theta$

$\displaystyle = \frac{1}{2}\int_0^{\frac{\pi}{4}} (1-\cos(2\theta))\, d\theta = \frac{1}{2}\left[\theta - \frac{1}{2}\sin(2\theta)\right]_0^{\frac{\pi}{4}}$

$\displaystyle = \frac{1}{2}\left[\left(\frac{\pi}{4} - \frac{1}{2}\sin\left(\frac{\pi}{2}\right)\right) - \left(0 - \frac{1}{2}\sin(0)\right)\right]$

$\displaystyle = \boxed{\frac{\pi}{8} - \frac{1}{4}}$