

Matthew's question: S 8.3 #35

Why use $x = r \cos \theta$ when the cheat sheet says $x = r \sin \theta$? The easy answer is phase change.

Book way: You end up with $\cos^{-1}(\cos \theta) = \theta$ in the final step.

Cheat Sheet way: You end up with $\frac{\pi}{2} - \sin^{-1}(\cos \theta)$ in the final step.

Notice that $\cos(\theta - \frac{\pi}{2}) = \sin \theta$:

$$\begin{aligned} \text{This gives } \frac{\pi}{2} - \sin^{-1}(\cos \theta) \\ = \frac{\pi}{2} - \sin^{-1}(\sin(\theta + \frac{\pi}{2})) \end{aligned}$$

Slight technical problem, here, because we're taking the argument of sine beyond its legal restriction for use of arcsine!

We CAN shift sine to the RIGHT $\frac{\pi}{2}$ and flip it, to get cosine:

$$\cos \theta = -\sin(\theta - \frac{\pi}{2}) = \sin(\frac{\pi}{2} - \theta) \quad (\text{See Trig Cheat in front of book})$$

$$\begin{aligned} \text{Then } \frac{\pi}{2} - \sin^{-1}(\cos \theta) &= \frac{\pi}{2} - \sin^{-1}(\sin(\frac{\pi}{2} - \theta)) \\ &= \frac{\pi}{2} - \frac{\pi}{2} + \theta = \theta \quad \checkmark \end{aligned}$$