202 §8.2 #s 6, 9, 12, 18, 27, 28, 37, 38, 44, 55, 62

#s 1-49 Evaluate the integral

(6) $\displaystyle\int \frac{\sin^3\sqrt{x}}{\sqrt{x}}\,dx$ Let $u=\sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}\,dx$

$$= 2\int \sin^3\sqrt{x}\cdot\frac{1}{2\sqrt{x}}\,dx = 2\int \sin^3 u\,du$$

$$= 2\int \sin^2 u \sin u\,du = 2\int (1-\cos^2 u)\sin u\,du$$

$$= 2\int (\sin u - \cos^2 u \sin u)\,du$$

Let $v = \cos u$

$dv = -\sin u\,du$

$$= 2\int \sin u\,du - 2\int \cos^2 u \sin u\,du$$

$$= -\left[-\cos(u)\right] + 2\int \cos^2 u(-\sin u\,du)$$

$$= \cos u + 2\int v^2\,dv = \cos u + 2\,\frac{v^3}{3} + C$$

$$= \cos\sqrt{x} + 2\,\frac{\cos^3 u}{3} + C = \boxed{\cos\sqrt{x} + \frac{2}{3}\cos^3\sqrt{x} + C}$$

(9) $\displaystyle\int_0^{\pi} \sin^4(3t)\,dt = \int_0^{\pi} \sin^2(3t)\sin^2(3t)\,dt$

$$= \frac{1}{2}\cdot\frac{1}{2}\int_0^{\pi} (1-\cos(6t))(1-\cos(6t))\,dt$$

$$= \frac{1}{4}\int_0^{\pi} (1 - 2\cos(6t) + \cos^2(6t))\,dt$$

202 §8.2 #s 9, 12, 18, 27, 28, 37, 38, 44, 55, 62

9 cnt'd

$$= \frac{1}{4}\int_0^{\pi} 1\,dt - \frac{1}{4}\int_0^{\pi} 2\cos(6t)\,dt + \frac{1}{4}\int_0^{\pi} \frac{1}{2}(1+\cos(12t))\,dt$$

$$= \frac{1}{4}\int_0^{\pi} dt - \frac{1}{2}\cdot\frac{1}{6}\int_0^{\pi}\cos(6t)\,6\,dt + \frac{1}{8}\int_0^{\pi} dt + \frac{1}{8}\int_0^{\pi}\cos(12t)\,dt$$

$$= \frac{1}{4}\pi - \frac{1}{12}\cos(6t)\Big|_0^{\pi} + \frac{1}{8}\pi + \frac{1}{8}\cdot\frac{1}{12}\int_0^{\pi}\cos(12t)\cdot 12\,dt$$

$$= \frac{3}{8}\pi - \frac{1}{12}\left[\cos(6\pi) - \cos(0)\right] + \frac{1}{96}\cos(12t)\Big|_0^{\pi}$$

$$= \boxed{\frac{3}{8}\pi}$$

12 $\displaystyle\int x\cos^2 x\,dx$

$$\left(\begin{array}{l} u = x \longrightarrow du = dx \\ dv = \cos^2 x\,dx = \frac{1}{2}(1+\cos(2x))\,dx \\ \Longrightarrow v = \frac{1}{2}x + \frac{1}{4}\sin(2x) \end{array}\right)$$

$$= uv - \int v\,du = x\left(\frac{1}{2}x + \frac{1}{4}\sin(2x)\right) - \int \frac{1}{2}x + \frac{1}{4}\sin(2x))\,dx$$

$$= \frac{1}{2}\left(x^2 + \frac{1}{2}x\sin(2x) - \left[\frac{1}{2}\cdot\frac{x^2}{2} - \frac{1}{8}\cos(2x)\right] + C\right.$$

$$= \frac{1}{2}x^2 + \frac{1}{4}x\sin(2x) - \frac{1}{4}x^2 + \frac{1}{8}\cos(2x) + C$$

$$= \boxed{\frac{1}{4}x^2 + \frac{1}{4}x\sin(2x) + \frac{1}{8}\cos(2x) + C}$$

202 §8.2 #s 18, 27, 28, 37, 38, 44, 55, 62

(18) $\int \cot^5\theta \sin^4\theta \, d\theta$

Scratch: $\dfrac{\cos^5\theta}{\sin^5\theta} \cdot \sin^4\theta = \dfrac{\cos^5\theta}{\sin\theta}$

$= \dfrac{\cos^4\theta}{\sin\theta} \cdot \cos\theta = \dfrac{(1-\sin^2\theta)^2}{\sin\theta} \cos\theta$

$= \dfrac{1 - 2\sin^2\theta + \sin^4\theta}{\sin\theta} \cos\theta = \dfrac{\cos\theta}{\sin\theta} - 2\sin\theta\cos\theta +$

$+ \sin^3\theta \cos\theta$

$$= \int \underbrace{\frac{\cos\theta \, d\theta}{\sin\theta}}_{\frac{du}{u}} - 2\int \underbrace{\sin\theta \cos\theta \, d\theta}_{u\,du} + \int \underbrace{\sin^3\theta \cos\theta \, d\theta}_{u^3 du}$$

$$= \ln|\sin\theta| - 2\frac{\sin^2\theta}{2} + \frac{\sin^4\theta}{4} + C$$

$$= \boxed{\ln|\sin\theta| - \sin^2\theta + \tfrac{1}{4}\sin^4\theta + C}$$

(27) $\displaystyle\int_0^{\frac{\pi}{3}} \tan^5 x \sec^4 x \, dx = \int_0^{\frac{\pi}{3}} \tan^5 x \,(1 + \tan^2 x) \sec^2 x \, dx$

$$= \int_0^{\frac{\pi}{3}} \tan^5 x \sec^2 x \, dx + \int_0^{\frac{\pi}{3}} \tan^7 x \sec^2 x \, dx$$

$$= \frac{1}{6}\tan^6 x \Big|_0^{\frac{\pi}{3}} + \frac{1}{8}\tan^8 x \Big|_0^{\frac{\pi}{3}} = \frac{1}{6}(\sqrt{3})^6 - 0 + \frac{1}{8}(\sqrt{3})^8 - 0$$

$$= \frac{27}{6} + \frac{81}{8} = \frac{9}{2} + \frac{81}{8} = \frac{36}{8} + \frac{81}{8} = \boxed{\frac{117}{8}}$$

202 § 8.2 #s 28, 37, 38, 44, 55, 62

(28) $\int \tan^3(2x)\sec^5(2x)\,dx$

$\tan^3(2x)\sec^5(2x) = \tan^2(2x)\sec^4(2x)\sec(2x)\tan(2x)$

$= (\sec^2(2x)-1)\sec^4(2x)\,(\sec(2x)\tan(2x))$

$= (\sec^6(2x)-\sec^4(2x))\sec(2x)\tan(2x)$

Let $u=\sec(2x)$. Then $\sec(2x)\tan(2x)\cdot 2\,dx = du$

$= \frac{1}{2}\int (u^6-u^4)\,du = \frac{1}{2}\left[\frac{1}{7}u^7-\frac{1}{5}u^5\right]+C$

$= \boxed{\frac{1}{14}\sec^7(2x)-\frac{1}{10}\sec^5(2x)+C}$

(37) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot^2 x\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\csc^2 x-1)\,dx = -\cot x\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} - x\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

1, 0, 2, 1, $\sqrt{3}$

$= -[0-\sqrt{3}] - \left[\frac{\pi}{2}-\frac{\pi}{6}\right] = \sqrt{3}-\frac{2\pi}{6} = \boxed{\sqrt{3}-\frac{1}{3}\pi}$

(38) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^3 x\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\csc^2 x-1)\cot x\,dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x\csc^2 x\,dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x\,dx = -\frac{1}{2}\cot^2 x\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \ln|\sin x|\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$u\cdot(-du)$ $\qquad \frac{\cos x}{\sin x}dx$

$= -\frac{1}{2}[0^2-1^2] - \left[\ln|\sin(\tfrac{\pi}{2})| - \ln|\sin(\tfrac{\pi}{4})|\right] = \frac{1}{2} - \left[\ln(1)-\ln(\tfrac{\sqrt{2}}{2})\right]$

$= \boxed{\frac{1}{2}+\ln(\tfrac{\sqrt{2}}{2})}$

202 §8.2 #s 44, 55, 62

44 $\int \cos(\pi x)\cos(4\pi x)\,dx$

(ID: $\cos(x+y) = \cos(x)\cos(y) - \sin x \sin y$

$\cos x \cos y = \cos(x+y) - \sin x \sin y$

No. Use the ID's pg 501:

$\cos A \cos B = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$)

By 2c, pg 501, we have

$\frac{1}{2}\int \left(\cos(\pi x - 4\pi x) + \cos(\pi x + 4\pi x)\right) dx$

$= \frac{1}{2}\int \left(\cos(-3\pi x) + \cos(5\pi x)\right) dx$

$= \frac{1}{2}\left[\frac{1}{3\pi}\sin(3\pi x) + \frac{1}{5\pi}\sin(5\pi x)\right] + C$

$= \boxed{\frac{1}{6\pi}\sin(3\pi x) + \frac{1}{10\pi}\sin(5\pi x) + C}$

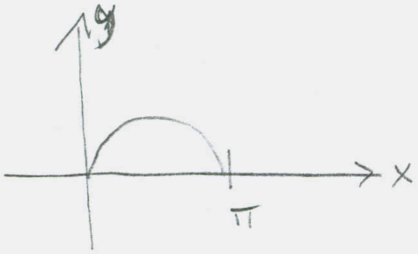
55 Find f_{AVG} for $f(x) = \sin^2 x \cos^3 x$ on $[-\pi, \pi]$.

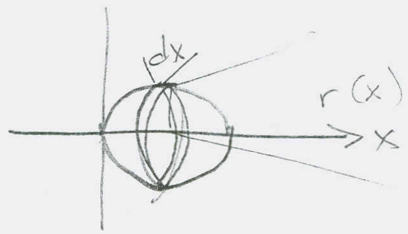
$\frac{1}{\pi-(-\pi)}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin^2 x \cos^3 x\,dx = \boxed{0}$ b/c f is ODD!

202 §8.2 #62

(62) Find the volume obtained by rotating the region bdd by the given curves about the specified axis.

$y = \sin^2 x$, $y = 0$, $0 \le x \le \pi$, x-axis





Disc: $V = \pi r^2 h = \pi (f(x))^2 dx$

$$V = \pi \int_0^\pi \sin^4(x)\,dx = \frac{\pi}{4}\int_0^\pi (1-\cos(2x))^2\,dx$$

$$= \frac{\pi}{4}\int_0^\pi \left(1 - 2\cos(2x) + \cos^2(2x)\right) dx$$

$$= \frac{\pi}{4}\int_0^\pi dx - \frac{\pi}{4}\int_0^\pi \cos(2x)\cdot 2\,dx + \frac{\pi}{4}\int_0^\pi \frac{1}{2}(1+\cos(4x))\,dx$$

$$= \frac{\pi}{4}[\pi] - \frac{\pi}{4}\left[\sin(2x)\right]_0^\pi + \frac{\pi}{8}\int_0^\pi dx + \frac{\pi}{8}\cdot\frac{1}{4}\int_0^\pi \cos(4x)\cdot 4\,dx$$

$$= \frac{\pi^2}{4} + \frac{\pi^2}{8} + \frac{\pi}{32}\left[\sin(8x)\right]_0^\pi = \frac{3\pi^2}{8}$$

Nothing in this assignment involving

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

SEE pg 500!