202 §8.1 #s 2, 4, 8, 10, 17, 24, 30, 33, 34

(2) Integrate by parts.

$$\int \theta \cos\theta \, d\theta \quad \left(\begin{array}{ll} u=\theta, & dv=\cos\theta\, d\theta \Rightarrow \\ du=d\theta & v=\sin\theta \end{array} \right)$$

2/3

$$= uv - \int v\,du = \theta \sin\theta - \int \sin\theta \, d\theta$$

$$= \boxed{\theta \sin\theta + \cos\theta + C}$$

#s 3-32 Evaluate the Integral

(4) $\int x e^{-x}\,dx$ $\left(\begin{array}{l} \text{Let } u=x \Rightarrow du=dx \\ dv=e^{-x}dx \Rightarrow v=-e^{-x} \end{array} \right)$

$$= uv - \int v\,du$$

$$= x(-e^{-x}) - \int -e^{-x}\,dx$$

2/3

$$= \boxed{-xe^{-x} - e^{-x} + C}$$

(8) $\int x^2 \cos(mx)\,dx$

Let $u = x^2 \Rightarrow du = 2x\,dx$

$dv = \cos(mx)\,dx \Rightarrow v = \frac{1}{m}\sin(mx)$

$$= uv - \int v\,du = x^2 \cdot \frac{1}{m}\sin(mx) - \int \frac{1}{m}\sin(mx)\cdot 2x\,dx$$

Let $u = x \Rightarrow du = dx$

$dv = \sin(mx)\,dx \Rightarrow v = \frac{-1}{m}\cos(mx)$

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8 cnt'd

$$= \frac{1}{m}x^2\sin(mx) - \frac{2}{m}\int x\sin(mx)\,dx$$

$$= \frac{1}{m}x^2\sin(mx) - \frac{2}{m}\left[uv - \int v\,du\right]$$

$$= \frac{1}{m}x^2\sin(mx) - \frac{2}{m}\left[x\left(-\frac{1}{m}\cos(mx)\right) - \int -\frac{1}{m}\cos(mx)\,dx\right]$$

$$= \frac{1}{m}x^2\sin(mx) - \frac{2}{m}\left[-\frac{1}{m}x\cos(mx) + \frac{1}{m}\cdot\frac{1}{m}\sin(mx)\right] + C$$

$$\boxed{= \frac{1}{m}x^2\sin(mx) + \frac{2}{m^2}x\cos(mx) - \frac{2}{m^3}\sin(mx) + C}$$

10 $\int \sin^{-1}(x)\,dx$

17 $\int e^{2\theta}\sin(3\theta)\,d\theta$

$u = e^{2\theta} \Longrightarrow du = 2e^{2\theta}d\theta$

$dv = \sin(3\theta)d\theta \Longrightarrow v = -\frac{1}{3}\cos(3\theta)$

$$= uv - \int v\,du = e^{2\theta}\left(-\frac{1}{3}\cos(3\theta)\right) - \int 2e^{2\theta}\left(-\frac{1}{3}\cos(3\theta)\,d\theta\right)$$

Let $u = e^{2\theta} \Longrightarrow du = 2e^{2\theta}d\theta$

$dv = \cos(3\theta)d\theta \Longrightarrow v = \frac{1}{3}\sin(3\theta)d\theta$

$$\cancel{= -\frac{1}{3}e^{2\theta}\cos(3\theta) - \left[uv - \int v\,du\right]}$$

$$\cancel{= -\frac{1}{3}e^{2\theta}\cos(3\theta) - \left[\frac{1}{3}e^{2\theta}\sin(3\theta) - \int \frac{1}{3}\left(\right.\right.}$$

202 §8.1 #s 17, 24, 30, 33, 34

(17) $= -\frac{1}{3}e^{2\theta}\cos(3\theta) + \frac{2}{3}\int e^{2\theta}\cos(3\theta)\,d\theta$

$= -\frac{1}{3}e^{2\theta}\cos(3\theta) + \frac{2}{3}\left[uv - \int v\,du\right]$

$= -\frac{1}{3}e^{2\theta}\cos(3\theta) + \frac{2}{3}\left[\frac{1}{3}e^{2\theta}\sin(3\theta) - \int \frac{2}{3}e^{2\theta}\sin(3\theta)\,d\theta\right]$

$= -\frac{1}{3}e^{2\theta}\cos(3\theta) + \frac{2}{9}e^{2\theta}\sin(3\theta) - \frac{4}{9}\int e^{2\theta}\sin(3\theta)\,d\theta$

$= \int e^{2\theta}\sin(3\theta)\,d\theta \longrightarrow$

$a = b + c + (-\frac{4}{9})a \longrightarrow$

$a = \frac{9}{13}(b+c)$

$\int e^{2\theta}\sin(3\theta)\,d\theta = \frac{9}{13}\left[\frac{1}{3}e^{2\theta}\cos(3\theta) + \frac{2}{9}e^{2\theta}\sin(3\theta)\right] + C$

(24) $\int_0^{\pi} x^3\cos(x)\,dx$

$u = x^3,\ du = 3x^2dx$

$dv = \cos(x)dx,\ v = \sin(x)$

$= uv - \int v\,du = x^3\sin(x)\Big]_0^{\pi} - \int_0^{\pi} 3x^2\sin(x)\,dx$

$u = x^2 \quad du = 2x\,dx$

$dv = \sin(x) \quad v = -\cos x$

2 pts

$= x^3\sin(x)\Big]_0^{\pi} - 3\left[uv - \int v\,du\right]$

$= x^3\sin(x)\Big]_0^{\pi} - 3\left[\left[-x^2\cos(x)\right]_0^{\pi} - \int_0^{\pi} 2x(-\cos(x))dx\right]$

$u = x \quad du = dx$

$dv = \cos(x)dx$

$v = \sin(x)$

$= 0 - 0 - 3\left[-\pi^2\cdot(-1) - 0\cdot 1\right] + 3\cdot 2\int_0^{\pi} x\cos(x)\,dx$

$= -3\pi^2 + 6\left[uv - \int v\,du\right] = -3\pi^2 + 6\left[\left[x\sin(x)\right]_0^{\pi} - \int_0^{\pi}\sin(x)\,dx\right]$

$= 3\pi^2 + 6\left[[0-0] + \cos(x)\right]_0^{\pi} = 3\pi^2 + 6\left[-1-(1)\right] =$

$= \boxed{3\pi^2 - 12}$

202 §8.1 #s 30, 33, 34

(30) $\displaystyle\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr$

Instinct suggests differentiating the r^3 to make it go away.

$u = r^3 \Rightarrow du = 3r^2 dr$

This requires an antiderivative for $\frac{1}{\sqrt{4+r^2}}$, which wouldn't be bad, if we had a $2r\,dr$ for it.

Hmmmm. Try *that*:

$u = r^2 \Rightarrow du = 2r\,dr$

$$dv = \frac{r\,dr}{\sqrt{4+r^2}} = \frac{1}{2}\cdot\frac{2r\,dr}{\sqrt{4+r^2}} = \frac{1}{2}\cdot\frac{ds}{\sqrt{s}}, \text{ where}$$

$s = r^2+4$. This is $\frac{1}{2}s^{-\frac{1}{2}}ds \Rightarrow$

$$v = \frac{1}{2}\cdot\frac{1}{\frac{1}{2}}s^{\frac{1}{2}} = s^{\frac{1}{2}} = \sqrt{r^2+4}$$ This will work!

$$uv - \int v\,du = r^2\cdot\sqrt{r^2+4}\Big]_0^1 - \int_0^1 \sqrt{r^2+4}\cdot 2r\,dr$$

$$= \left[1^2\sqrt{1^2+4} - 0^2\sqrt{0^2+4}\right] - \frac{2}{3}(r^2+4)^{\frac{3}{2}}\Big]_0^1,$$ since this is of the form $\int u^{\frac{1}{2}}du = \frac{2}{3}u^{\frac{3}{2}}+C$. This gives

$$\sqrt{5} - \frac{2}{3}\left[(1^2+4)^{\frac{3}{2}} - (0^2+4)^{\frac{3}{2}}\right] = \sqrt{5} - \frac{2}{3}\left[5^{\frac{3}{2}} - 4^{\frac{3}{2}}\right]$$

$$= \sqrt{5} - \frac{2}{3}\left[5\sqrt{5} - 8\right] = \sqrt{5} - \frac{10}{3}\sqrt{5} + \frac{16}{3}$$

$$= \boxed{\frac{16}{3} - \frac{7}{3}\sqrt{5}}$$

202 §8.2 #s 33, 34

#s 33–38 Make a substitution and use integration by parts

33 $\int \cos\sqrt{x}\, dx$

Let $y = \sqrt{x}$. Then

$dy = \frac{1}{2}x^{-\frac{1}{2}}dx = \frac{1}{2\sqrt{x}}dx \Rightarrow$

$dx = 2\sqrt{x}\, dy = 2y\, dy$

This gives:

$$\int \cos(y)\cdot 2y\, dy = 2\int y\cos(y)\, dy$$

$$\left(\begin{array}{l} \text{Let } u = y \Rightarrow du = dy \\ dv = \cos(y)dy \Rightarrow v = \sin(y) \end{array}\right)$$

$$= uv - \int v\, du = y\sin(y) - \int \sin(y)\, dy$$

$$= y\sin(y) + \cos(y) + C$$

$$\boxed{= \sqrt{x}\sin\sqrt{x} + \cos\sqrt{x} + C}$$

34 $\int t^3 e^{-t^2} dt$

28/3

$y = -t^2 \Rightarrow dy = -2t\, dt$

$\Rightarrow dt = \frac{dy}{-2t}$

$$= \int t^3 e^y \cdot \frac{dy}{-2t} = -\frac{1}{2}\int e^y \cdot t^2 dy = \frac{1}{2}\int e^y y\, dy$$

$$= \frac{1}{2}\int y e^y dy = \frac{1}{2}\left[ye^y - \int e^y dy\right] = \frac{1}{2}(-t^2)e^{-t^2} - e^{-t^2} + C$$

$$= \boxed{-\frac{1}{2}t^2e^{-t^2} - e^{-t^2} + C}$$