202 §7.8 #s 1a-e, 6, 10, 17, 20, 32, 44, 60, 64, 72, 85

(1) Given: $\lim_{x\to a} f(x) = 0$, $\lim_{x\to a} g(x) = 0$, $\lim_{x\to a} h(x) = 1$

$\lim_{x\to a} p(x) = \infty$, $\lim_{x\to a} q(x) = \infty$.

Which of the following are indeterminate forms? For those that are not, evaluate the limit, where possible.

"Yes"

(a) $\lim_{x\to a} \frac{f(x)}{g(x)}$ Yes

(b) $\lim_{x\to a} \frac{f(x)}{p(x)} = 0$

(c) $\lim_{x\to a} \frac{h(x)}{p(x)} = 0$

(d) $\lim_{x\to a} \frac{p(x)}{f(x)} = \infty, -\infty$ or $\nexists$.

(e) $\lim_{x\to a} \frac{p(x)}{q(x)}$ Yes.

(6) $\lim_{x\to 2} \frac{x^2+x-6}{x-2} = \lim_{x\to 2} \frac{(x+3)(x-2)}{x-2} = \lim_{x\to 2} (x+3) = 5$

#s 5–64 Use L'Hopital's Rule to evaluate the limits

(10) $\lim_{x\to 0} \frac{\sin(4x)}{\tan(5x)} \overset{L'H}{=} \lim_{x\to 0} \frac{4\cos(4x)}{5\sec^2(5x)} = \frac{4}{5} \lim_{x\to 0} \cos(4x)\cos(5x) = \boxed{\frac{4}{5}}$

202 §7.8 #s 17, 20, 32, 44, 60, 64, 72, 85

(17) $\lim_{x \to 0^+} \frac{\ln(x)}{x} = \infty$

$\frac{\infty}{0}$

(20) $\lim_{x \to 1} \frac{\ln(x)}{\sin(\pi x)} \overset{L'H}{=} \lim_{x \to 1} \frac{\frac{1}{x}}{\pi\cos(\pi x)} = \frac{\frac{1}{1}}{\pi \cos(\pi)} = \boxed{-\frac{1}{\pi}}$

$\frac{0}{0}$

(32) $\lim_{x \to 0} \frac{x}{\arctan(4x)} \overset{L'H}{=} \lim_{x \to 0} \frac{1}{\frac{1}{x^2+1}} = \lim_{x \to 0} (x^2+1) = \boxed{1}$

$\frac{0}{0}$

(44) $\lim_{x \to \frac{\pi}{4}} \left((1-\tan(x))(\sec(x))\right) = \boxed{0}$

$0 \cdot \sqrt{2}$

(60) $\lim_{x \to \infty} (e^x + x)^{\frac{1}{x}} = \lim_{x \to \infty} y \implies$

$\lim_{x \to \infty} (\ln(y)) = \lim_{x \to \infty} \left(\frac{1}{x} \ln(e^x + x)\right) = \lim_{x \to \infty} \frac{\ln(e^x+x)}{x}$

$0 \cdot \infty$ $\qquad \frac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x \to \infty} \frac{\frac{e^x+1}{e^x+x}}{1} = \lim_{x \to \infty} \frac{e^x+1}{e^x+x} \overset{L'H}{=} \lim_{x \to \infty} \frac{e^x}{e^x+1} = 1 = \ln(y)$

$\implies \lim_{x \to \infty} y = e^1 = \boxed{e}$

202 §7.8 #s 64, 72, 85

64 Asked in class. I'll try to write it up nicely.

$$\lim_{x\to\infty}\left(\left(\frac{2x-3}{2x+5}\right)^{2x+1}\right) = \lim_{x\to\infty} y \Longrightarrow$$

$$\lim_{x\to\infty}(\ln(y)) = \lim_{x\to\infty}\left((2x+1)\ln\left(\frac{2x-3}{2x+5}\right)\right)$$

$$\infty \cdot 0$$

$$= \lim_{x\to\infty}\left((2x+1)(\ln(2x-3)-\ln(2x+5))\right)$$

$$= \lim_{x\to\infty}\frac{\ln(2x-3)-\ln(2x+5)}{(2x+1)^{-1}} \overset{L'H}{=} \lim_{x\to\infty}\frac{\frac{2}{2x-3}-\frac{2}{2x+5}}{-(2x+1)^{-2}(2)}$$

$$= \lim_{x\to\infty}\frac{2\left(\frac{1}{2x-3}-\frac{1}{2x+5}\right)}{2\left(\frac{1}{(2x+1)^2}\right)} = \lim_{x\to\infty}\left(\left(\frac{2x+5-(2x-3)}{(2x+5)(2x-3)}\right)\left((2x+1)^2\right)\right)$$

$$= \lim_{x\to\infty}\left(\frac{8(4x^2+4x+1)}{4x^2+4x-15}\right) = 8\lim_{x\to\infty}\frac{4x^2+4x+1}{4x^2+4x-5} = 8$$

$$\therefore \lim_{x\to\infty} y = \boxed{e^8}$$

Showed a little more than in class.

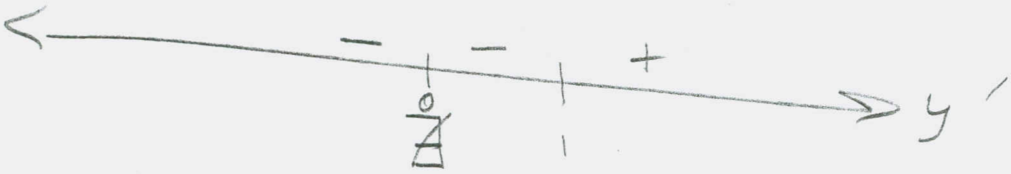
202 §7.8 #s 72, 85

72 Sketch the curve

$$y = \frac{e^x}{x} \qquad \mathcal{D} = \mathbb{R} \setminus \{0\} \qquad f(1) = e$$

V.A.: $x = 1$

$$y' = \frac{e^x \cdot x - e^x \cdot 1}{x^2} = \frac{x-1}{x^2} e^x \underset{=}{\text{SET}} 0 \Longrightarrow x = 1$$



$$y'' = \frac{x^2 - (x-1)(2x)}{x^4} e^x + \frac{x-1}{x^2} e^x$$

$$= \frac{x^2 - 2x^2 + 2x}{x^4} e^x + \frac{x-1}{x^2} e^x$$

$$= \frac{x^2 - 2x^2 + 2x + x^3 - x^2}{x^4} e^x = \frac{x^3 - 2x^2 + 2x}{x^4} e^x$$

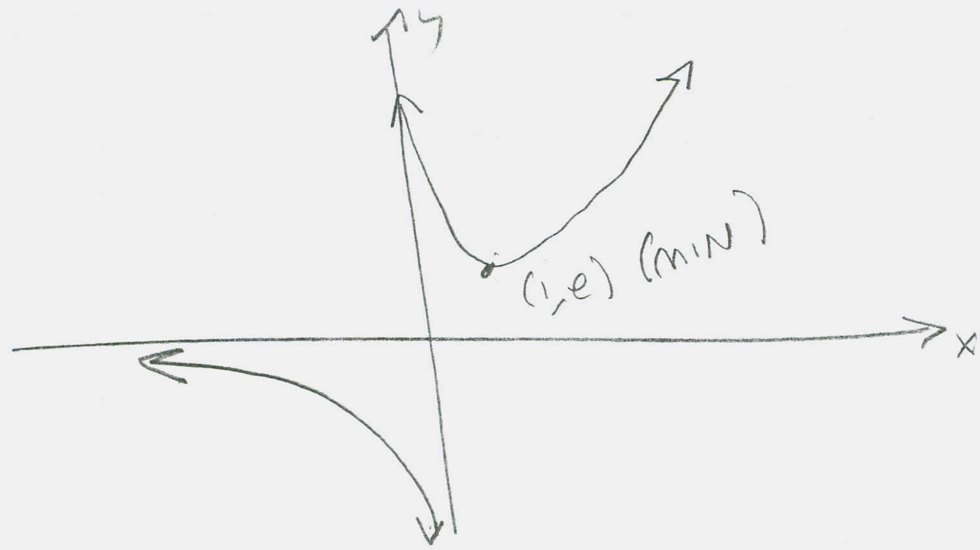
$$= \frac{x^2 - 2x + 2}{x^3} e^x \underset{=}{\text{SET}} 0 \Longrightarrow x^2 - 2x + 2 = 0$$

$$\Longrightarrow x^2 - 2x + 1^2 = -2 + 1^2$$

$(x-1)^2 = -1 \Longrightarrow$ No Real Sol'n.

$\therefore$ $f'' > 0$ on $(-\infty, 0)$
$f'' > 0$ on $(0, \infty)$ } Check both sides of the V.A. $x = 0$.

202 §78 #s 72, 85




(85) We prove, in essence, that $e = \lim_{n\to\infty}\left(1+\frac{r}{n}\right)^n = e^r$

with L'Hopital:

$$A = A_0\left(1+\frac{r}{n}\right)^{nt}$$

Let $y = \left(1+\frac{r}{n}\right)^n$. Then $\ln(y) = \ln\left(\left(1+\frac{r}{n}\right)^n\right)$

$$= n\ln\left(1+\frac{r}{n}\right) \Longrightarrow$$

$$\lim_{n\to\infty}(\ln(y)) = \lim_{n\to\infty}\left(n\ln\left(1+\frac{r}{n}\right)\right) = \lim_{n\to\infty}\left(\frac{\ln\left(1+\frac{r}{n}\right)}{\frac{1}{n}}\right)$$

L'H

$$= \lim_{n\to\infty}\left(\frac{\left(\frac{-\frac{r}{n^2}}{1+\frac{r}{n}}\right)}{-\frac{1}{n^2}}\right) = \lim_{n\to\infty}\left(\frac{\left(-\frac{r}{n^2}\right)\left(-\frac{n^2}{1}\right)}{1+\frac{r}{n}}\right) = \frac{r}{1} = r$$

$$\Longrightarrow \lim_{n\to\infty}(y) = e^r \Longrightarrow A_0\left(1+\frac{r}{n}\right)^{nt} = A_0e^{rt}!$$

$\blacksquare$