202 §7.7 #s 1-6, 8, 10, 18, 19, 20, 24, 26

$\sinh^{-1}(x) = \ln(x + \sqrt{x^2+1})$

$\cosh^{-1}(x) = \ln(x + \sqrt{x^2-1})$, $x \geq 1$

$\tanh^{-1}(x) = \ln\left(\frac{1+x}{1-x}\right)$, $-1 < x < 1$

#s 1-6 Find the numerical value

① (a) $\sinh(0) = \frac{e^0 - e^{-0}}{2} = 0$

(b) $\cosh(0) = \frac{1+1}{2} = 1$

② (a) $\tanh(0) = 0$

(b) $\tanh(1) = \frac{\sinh(1)}{\cosh(1)} = \frac{e^1 - e^{-1}}{e^1 + e^{-1}} = \frac{\frac{e^2-1}{e}}{\frac{e^2+1}{e}} = \frac{e^2-1}{e^2+1} \approx \boxed{.76159}$

③ (a) $\sinh(\ln(2))$

$= \frac{e^{\ln(2)} - e^{-\ln(2)}}{2}$

$= \frac{2 - \frac{1}{2}}{2}$

$= \frac{\frac{3}{2}}{2} = \boxed{\frac{3}{4}}$

(b) $\sinh(2)$

$= \frac{e^2 - e^{-2}}{2} = \frac{e^4 - 1}{2e^2} \approx \boxed{3.62686}$

④ (a) $\cosh(3)$

$= \frac{e^3 + e^{-3}}{2} \approx \boxed{10.06766}$

(b) $\cosh(\ln(3))$

$= \frac{e^{\ln(3)} + e^{-\ln(3)}}{2}$

$= \frac{3 + \frac{1}{3}}{2} = \frac{\frac{4}{3}}{2} = \boxed{\frac{2}{3}}$

⑤ (a) $\operatorname{sech}(0) = \boxed{1}$

(b) $\cosh^{-1}(1)$

$= \ln(1 + \sqrt{1^2 - 1})$

$= \boxed{0}$

⑥ (a) $\sinh(1)$

$= \frac{e - \frac{1}{e}}{2} \approx \boxed{1.17520}$

(b) $\sinh^{-1}(1) = \ln(1 + \sqrt{1^2+1})$

$= \ln(1 + \sqrt{2})$

$\approx \boxed{.88137}$

202 §7.7 #s 8, 10, 18, 19, 20, 24, 26

#s 7–19 Prove the identity

8 $\cosh(-x) = \cosh(x)$

Pf $\cosh(-x) = \dfrac{e^{-x}+e^{-(-x)}}{2} = \dfrac{e^{x}+e^{-x}}{2} = \cosh(x)$

10 $\cosh(x) - \sinh(x) = e^{-x}$

Pf $\dfrac{e^x+e^{-x}}{2} - \dfrac{e^x-e^{-x}}{2} = \dfrac{e^x+e^{-x}-e^x+e^{-x}}{2} = e^{-x}$ ✓

18 $\dfrac{1+\tanh x}{1-\tanh x} = e^{2x}$

Pf $\dfrac{\dfrac{\cosh(x)+\sinh(x)}{\cosh(x)}}{\dfrac{\cosh(x)-\sinh(x)}{\cosh(x)}} = \dfrac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)} = \dfrac{e^x}{e^{-x}} = \boxed{e^{2x}}$

19 $(\cosh(x)+\sinh(x))^n = \cosh(nx)+\sinh(nx)$

Pf $(\cosh(x)+\sinh(x))^n = (e^x)^n = e^{nx} = \cosh(nx)+\sinh(nx)$ ☒

20 If $\tanh(x) = \frac{12}{13}$, find the values of the other hyperbolic functions.

$\operatorname{sech}^2(x) = 1 - \tanh^2(x) = 1 - \frac{144}{169} = \frac{169-144}{169} = \frac{25}{169} \Longrightarrow$

$\operatorname{sech}(x) = \pm\sqrt{\frac{25}{169}} = \pm\frac{5}{13}$, but $\operatorname{sech}(x) > 0$, so $+\frac{5}{13}$

$\boxed{\operatorname{sech}(x) = \frac{5}{13}} \Longrightarrow \boxed{\cosh(x) = \frac{13}{5}}$

$\dfrac{\sinh(x)}{\cosh(x)} = \dfrac{12}{13} = \dfrac{\sinh(x)}{\frac{13}{5}} \Longrightarrow \boxed{\frac{12}{5} = \sinh(x)} \longrightarrow \boxed{\frac{5}{12} = \operatorname{csch}(x)}$

$\boxed{\coth(x) = \frac{13}{12}}$

202 S'' 7.7 #s 24, 26

(24) Prove formulas for derivatives of

(a) $\cosh(x)$

(b) $\tanh(x)$

(c) $\operatorname{csch}(x)$

(d) $\operatorname{sech}(x)$

(e) $\coth(x)$

(a) $\frac{d}{dx}\left[\cosh(x)\right] = \sinh(x)$

Pf $\frac{d}{dx}\left[\frac{e^x+e^{-x}}{2}\right] = \frac{e^x-e^{-x}}{2} = \sinh(x)$ $\blacksquare$

(b) $\frac{d}{dx}\left[\tanh(x)\right] = -\operatorname{sech}^2(x)$

Pf $\frac{d}{dx}\left[\frac{\sinh(x)}{\cosh(x)}\right] = \frac{\cosh(x)\cosh(x) - \sinh(x)\sinh(x)}{\cosh^2(x)}$

$= \frac{\cosh^2(x) - \sinh^2(x)}{\cosh^2(x)} = \frac{1}{\cosh^2(x)} = \operatorname{sech}^2(x)$ $\blacksquare$

(c) $\frac{d}{dx}\left[\operatorname{csch}(x)\right] = -\operatorname{csch}(x)\coth(x)$

Pf $\frac{d}{dx}\left[\frac{1}{\sinh(x)}\right] = -\sinh(x)^{-2} \cdot \cosh(x) = -(\sinh(x))^{-1}\frac{\cosh(x)}{\sinh(x)}$

$= -\frac{1}{\sinh(x)} \cdot \coth(x) = -\operatorname{csch}(x)\coth(x)$ $\blacksquare$

202 §7.7 #s 24, 26

(24) Cnt'd

(d) $\frac{d}{dx}\left[\operatorname{sech}(x)\right] = -\operatorname{sech}(x)\tanh(x)$

Pf

$$\frac{d}{dx}\left[\cosh(x)^{-1}\right] = -\cosh(x)^{-2}\cdot\sinh(x)$$

$$= -\frac{1}{\cosh(x)}\cdot\frac{\sinh(x)}{\cosh(x)} = -\operatorname{sech}(x)\tanh(x)$$

(e) $\frac{d}{dx}\left[\coth(x)\right] = -\operatorname{csch}^2(x)$

$\boxed{\text{Pf}}$

$$\frac{d}{dx}\left[\tanh(x)^{-1}\right] = -1\tanh(x)^{-2}\cdot\operatorname{sech}^2(x)$$

$$= -\frac{\cosh^2(x)}{\sinh^2(x)}\cdot\frac{1}{\cosh^2(x)} = -\frac{1}{\sinh^2(x)} = -\operatorname{csch}^2(x) \blacksquare$$

(26) Prove Equation 4

$$\cosh^{-1}(x) = \ln\left(x+\sqrt{x^2-1}\right)$$

$y = \cosh(x) = \frac{e^x+e^{-x}}{2}$

$x = \frac{e^y+e^{-y}}{2} \Longrightarrow$

$2x = e^y+e^{-y} \longrightarrow$

$2xe^y = e^{2y}+1$

$\Longrightarrow e^{2y}-2xe^y+1=0$

$b^2-4ac = (-2x)^2-4(1)(1)$
$= 4x^2-4$

$e^y = \frac{2x\pm\sqrt{4x^2-4}}{2}$

$= \frac{2\left(x\pm\sqrt{x^2-1}\right)}{2} = x\pm\sqrt{x^2-1}$

$\Longrightarrow y = \ln\left(x\pm\sqrt{x^2-1}\right)$. Almost.
How to argue that it's "+" and not "−"?

202 §7.7 #26

We have

$\cosh^{-1}(x) = \ln(x \pm \sqrt{x^2-1})$ and want to argue that it's "+" and not "−" in the $\pm$.

$y = \cosh^{-1}(x)$

$\cosh(y) = x$ From $\cosh(0) = 1$, we have

$\cosh^{-1}(1) = 0 \Longrightarrow$

$\ln(x \pm \sqrt{x^2-1}) = 0$ when $x = 1$

$\ln(1 \pm \sqrt{1^2-1}) = \begin{cases} 0 & \text{when we use "+"} \\ \nexists & \text{when we use "−"!} \end{cases}$

That's plenty of proof for our purposes $\blacksquare$