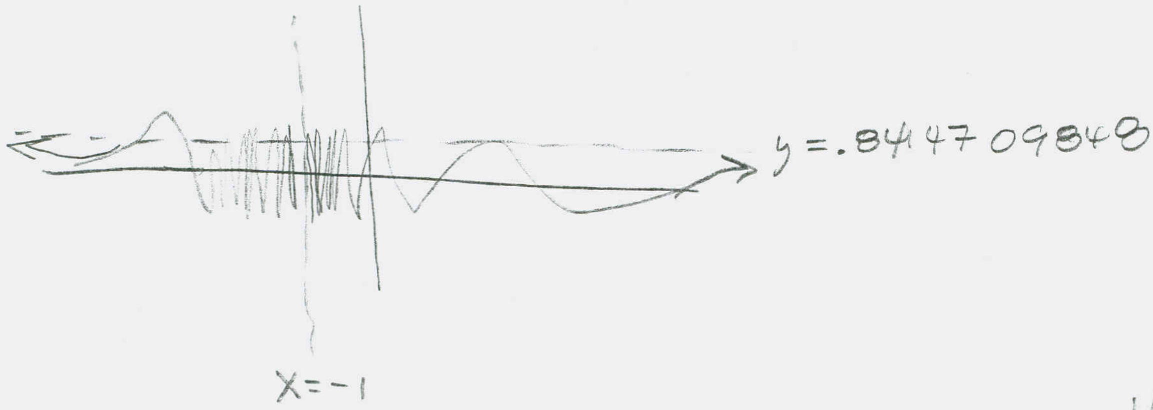
202 §7.6 II #s 51, 58, 60, 64, 70

(51) Sketch.

$y = \sin^{-1}\left(\frac{x}{x+1}\right) \quad \mathcal{D} = \mathbb{R} \setminus \{-1\}$

It oscillates infinitely often in a neighbor-hood of $x = -1$



(58) Find the antiderivative of $f'(x) = \frac{4}{\sqrt{1-x^2}}$,

given $f(\frac{1}{2}) = 1$

$$\int \frac{4\,dx}{\sqrt{1-x^2}} = 4\int \frac{dx}{\sqrt{1-x^2}} = \arcsin(x) + C = f(x)$$

$$f(\tfrac{1}{2}) = \arcsin(\tfrac{1}{2}) + C = \frac{\pi}{6} + C = 1 \longrightarrow C = 1 - \frac{\pi}{6}$$

$$\longrightarrow \boxed{f(x) = \arcsin(x) + 1 - \frac{\pi}{6}}$$

202 §7.6 II #s 60, 64, 70

#s 59–70 Evaluate the integral

(60) $\displaystyle\int \frac{\tan^{-1}(x)}{1+x^2}\,dx$

Let $u = \arctan(x)$.

Then $du = \frac{1}{1+x^2}\,dx$

$$= \int u\,du = \frac{u^2}{2} + C = \boxed{\frac{1}{2}\left(\arctan(x)\right)^2 + C}$$

(64) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin(x)}{1+\cos^2(x)}\,dx$

Let $u = \cos(x)$

Then $du = -\sin(x)\,dx$

$x = 0 \Rightarrow u = \cos(0) = 1$

$x = \frac{\pi}{2} \Rightarrow u = \cos(\frac{\pi}{2}) = 0$

$$= -\int_1^0 \frac{du}{1+u^2} = \int_0^1 \frac{du}{1+u^2}$$

$$= \arctan(u)\Big|_0^1 = \frac{\pi}{4} - 0 = \boxed{\frac{\pi}{4}}$$

(70) $\displaystyle\int \frac{x}{1+x^4}\,dx$

Let $u = x^2$

Then $du = 2x\,dx$

$$= \frac{1}{2}\int \frac{2x\,dx}{1+(x^2)^2} = \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\arctan(u) + C$$

$$= \boxed{\frac{1}{2}\arctan(x^2) + C}$$