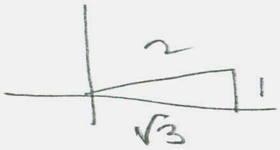
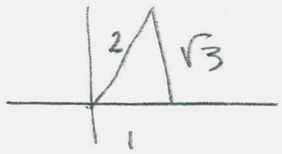
202 §7.6 #s 2, 5, 8, 14, 16, 18, 21, 28, 30, 44

#s 1-10 Find the exact value

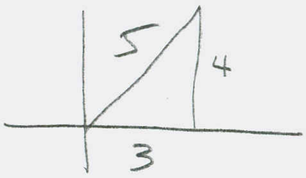
(2) (a) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \boxed{\frac{\pi}{6}}$ (b) $\sec^{-1}(2) = \boxed{\frac{\pi}{3}}$





(5) (a) $\tan(\arctan(10)) = 10$ (b) $\sin^{-1}\left(\sin\left(\frac{7\pi}{3}\right)\right) = \boxed{\frac{7\pi}{3}}$

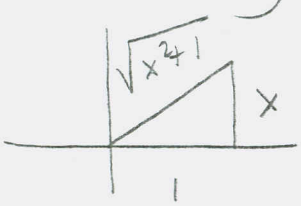
(8) $\csc\left(\arccos\left(\frac{3}{5}\right)\right) = \frac{5}{4}$



$5^2 - 3^2 = 16 = 4^2$

(14) Simplify $\cos(2\tan^{-1}x) = 2\sin(\tan^{-1}(x))\cos(\tan^{-1}(x))$

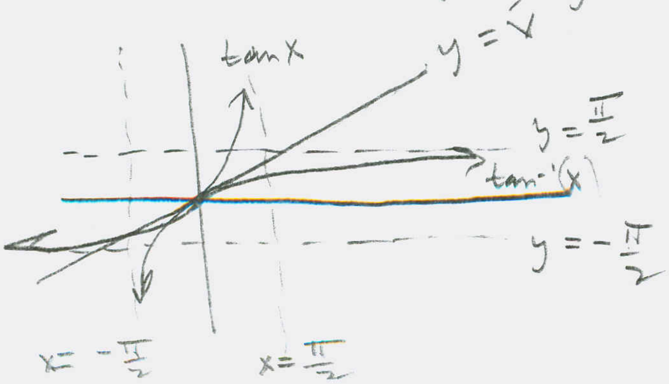
$= 2 \cdot \frac{x}{\sqrt{x^2+1}} \cdot \frac{1}{\sqrt{x^2+1}} = \boxed{\frac{2x}{x^2+1}}$



(16) Graph $\tan(x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and

$\tan^{-1}(x)$, $y = x$ on the same screen

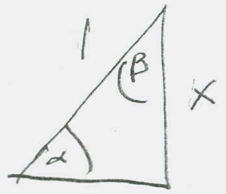
Not pretty, Steve.



202 § 7.6 #s 18, 21, 28, 30, 44

18 (a)

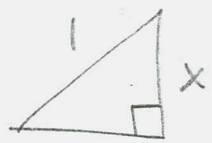
Prove that $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$



By picture:

$\sin^{-1}(x)$ is the angle α

$\sin^{-1}(x)$:



$\cos^{-1}(x)$ is the angle β and they're complementary.

Because $\sin^{-1}(x)$ and $\cos^{-1}(x)$ are both involved, our mutual/shared domain is $[0, \frac{\pi}{2}]$

Written Proof:

$\alpha = \sin^{-1}(x)$, $\beta = \cos^{-1}(x)$

$\cos(\alpha) = \cos(\sin^{-1}(x)) = \sqrt{1 - \sin^2(\sin^{-1}(x))}$ $*_1$

$= \sqrt{1 - x^2}$

$\sin(\beta) = \sin(\cos^{-1}(x)) = \sqrt{1 - \cos^2(\cos^{-1}(x))}$ $*_2$

$= \sqrt{1-x^2}$

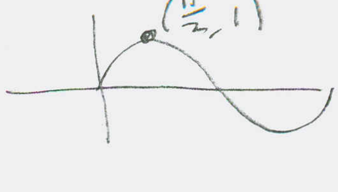
Now $\sin\left(\sin^{-1}(x) + \cos^{-1}(x)\right) = \sin(\alpha + \beta)$

$= \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$= x \cdot x + \sqrt{1-x^2}\sqrt{1-x^2}$

$= x^2 + 1 - x^2 = 1$, i.e., $\sin(\sin^{-1}(x) + \cos^{-1}(x)) = 1$,

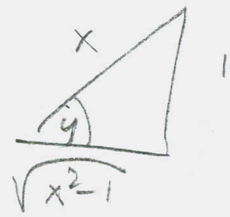
which means $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$.



202 §7.6 #s 21, 28, 30, 44

(21) $\frac{d}{dx}\left[\csc^{-1}(x)\right] = -\frac{1}{x\sqrt{x^2-1}}$

Pf $\quad y = \csc^{-1}(x) \Longrightarrow \csc(y) = x \Longrightarrow$



$-\csc(y)\cot(y)\frac{dy}{dx} = 1 \Longrightarrow$

$\frac{dy}{dx} = -\sin(y)\tan(y) = -\frac{1}{x}\cdot\frac{1}{\sqrt{x^2-1}}$ ■

#s 22-35 Differentiate

(28) $F(\theta) = \arcsin\sqrt{\sin\theta}$

$\Longrightarrow F'(\theta) = \frac{1}{\sqrt{1-\sin\theta}}\cdot\cos(\theta)$

(30) $y = \arctan\sqrt{\frac{1-x}{1+x}} \Longrightarrow$

$y' = \frac{1}{\frac{1-x}{1+x}+1}\cdot\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}}\cdot\frac{-1(x+1)-(1-x)(1)}{(x+1)^2}$

$= \frac{1}{\frac{1-x+1+x}{1+x}}\cdot\frac{1}{2\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}}\cdot\frac{-x-1-1+x}{(x+1)^2}$

$= \frac{x+1}{2}\cdot\frac{1}{2}\cdot\frac{\sqrt{1+x}}{\sqrt{1-x}}\cdot\frac{-2}{(x+1)^2} = \frac{-\sqrt{x+1}\,(x+1)}{2(x+1)^2\sqrt{1-x}}$

$= \frac{-1}{2\sqrt{x+1}\sqrt{1-x}} = \frac{-1}{2\sqrt{(1+x)(1-x)}} = \frac{-1}{2\sqrt{1-x^2}}$

202 §7.6 #44

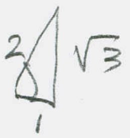
44 Find the limit

$$\lim_{x\to\infty} \arccos\left(\frac{1+x^2}{1+2x^2}\right)$$

Check $\lim_{x\to\infty} \frac{1+x^2}{1+2x^2} = \frac{1}{2}$ & arccos(x) is cnt's @ $x = \frac{1}{2}$, so we can move the limit inside:

$$\lim_{x\to\infty} \arccos\left(\frac{1+x^2}{1+2x^2}\right) = \arccos\left(\lim_{x\to\infty} \frac{1+x^2}{1+2x^2}\right)$$

$$= \arccos\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{3}}$$



202 §7.6 #s 18, 21, 28, 30, 44

18b Use (a) to prove $\frac{d}{dx}\left[\cos^{-1}(x)\right] = \frac{-1}{\sqrt{1-x^2}}$ $(-1 < x < 1)$

Proof Mean trick.

$$\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2} \implies$$

$$\frac{1}{\sqrt{1-x^2}} + \frac{d}{dx}\left[\cos^{-1}(x)\right] = 0 \implies$$

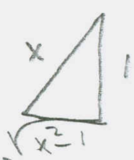
$\frac{d}{dx}\left[\cos^{-1}(x)\right] = -\frac{1}{\sqrt{1-x^2}}$, using formula for

$\frac{d}{dx}\left[\sin^{-1}(x)\right]$, which was already proved $\blacksquare$

21 Prove $\frac{d}{dx}\left[\csc^{-1}(x)\right] = -\frac{1}{x\sqrt{x^2-1}}$

Proof Let $y = \csc^{-1}(x) \implies$

$$\csc(y) = x \implies -\csc(y)\cot(y)\frac{dy}{dx} = 1 \implies$$



$$\frac{dy}{dx} = -\sin(y)\tan(y) = -\sin(\csc^{-1}(x))\tan(\csc^{-1}(x))$$

$$= -\frac{1}{x}\cdot\frac{1}{\sqrt{x^2-1}} = -\frac{1}{x\sqrt{x^2-1}} \quad \blacksquare$$

202 §7.6 #s 28, 30, 44

#s 22-35 Find derivative. Simplify.

(28) $F(\theta) = \arcsin(\sqrt{\sin\theta})$

$$\Rightarrow F'(\theta) = \frac{1}{\sqrt{1-\sin\theta}} \cdot \frac{1}{2}(\sin\theta)^{-\frac{1}{2}} \cdot \cos\theta$$

$$= \frac{\cos\theta}{2\sqrt{\sin\theta - \sin^2\theta}}$$

(30) $y = \arctan\left(\sqrt{\frac{1-x}{1+x}}\right)$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+\frac{1-x}{1+x}} \cdot \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}} \cdot \frac{(-1)(x+1)-(1-x)(1)}{(1+x)^2}$$

$$= \left(\frac{1}{\frac{1+x+1-x}{1+x}}\right)\left(\frac{1+x}{2(1-x)}\right)\left(\frac{-x-1-1+x}{(x+1)^2}\right)$$

$$= \left(\frac{x+1}{2}\right)\left(\frac{\sqrt{x+1}}{2\sqrt{(1-x)}}\right)\left(\frac{-2}{(x+1)^2}\right) = \frac{-1}{2\sqrt{1-x}\sqrt{1+x}} = \boxed{-\frac{1}{2\sqrt{1-x^2}}}$$

(44) $\lim_{x\to\infty} \arccos\left(\frac{x^2+1}{2x^2+1}\right) = \lim_{u\to\frac{1}{2}} (\arccos(\tfrac{1}{2})) = \boxed{\frac{\pi}{3}}$