202 §7.5 #s 1, 4a-e, 6abc, 8abc, 11, 13ab, 18ab

① Relative Growth rate is .7944 per member per day.
Let $N(t)=N_0e^{kt}$ model the pop. Given $N_0=2$,
Find $N(6)$ = pop. in 6 days:

$$N(t)=2e^{.7944t} \Longrightarrow N(6)=2e^{(.7944)(6)} \approx 234.991$$

$\approx$ 235 protozoa in the pop.

④ A bacteria pop has constant relative growth rate. After 2 hrs, there are 600 bacteria and after 8 hrs, the count is 75,000.

Matthew wants us to time-travel on this one.
I have my own methods, but I do like his way.

$N(t)=N_0e^{kt}$

$$\left.\begin{aligned} N(2)&=600=N_0e^{2k}\\ N(8)&=75000=N_0e^{8k}\end{aligned}\right\}\text{Given.}$$

① $$\frac{N(8)}{N(2)}=\frac{75000}{600}=\frac{e^{8k}}{e^{2k}}=e^{6k}$$

$$125=e^{6k}$$

$$6k=\ln(125)$$

$$k=\frac{\ln(125)}{6}=\frac{\ln(5^3)}{6}=\frac{3\ln(5)}{6}=\boxed{\frac{1}{2}\ln(5)}=\ln(\sqrt{5})$$

$$N(2)=N_0e^{2k}=N_0e^{2(\frac{1}{2}\ln(5))}=N_0e^{\ln(5)}=5N_0=600$$

$\Longrightarrow$ ⓐ $\boxed{N_0=\text{Initial pop}=120}$

ⓑ $\boxed{N(t)=120e^{\frac{1}{2}\ln(5)t} \quad \text{OR} \quad 120\cdot\sqrt{5}^{\,t}}$

202 §7.5 #s 4a-e, 6abc, 8abc, 11, 13ab, 18ab

4 cnt'd

c The # of cells after 5 hours is

$N(5) = 120e^{(\frac{1}{2}\ln(5))(5)}$

$= 120\sqrt{5}^5 \approx 6708.2 \approx \boxed{6708 = N(5)}$

d Rate of growth after 5 hours:

$N'(t) = 120 \cdot \frac{1}{2}\ln(5)\, e^{\frac{1}{2}\ln(5)t} = N'(t)$

$= 120\ln(\sqrt{5}) \cdot \sqrt{5}^t$

$N'(5) = 120\ln(\sqrt{5}) \cdot \sqrt{5}^5 \approx \boxed{647{,}7.86.264 \approx N'(5)}$

e When is pop = 200,000?

$N(t) = 120\sqrt{5}^t = 200000$

$\sqrt{5}^t = \frac{20000}{12} = \frac{5000}{3} = 1666.\overline{6}$

$\ln(\sqrt{5})t = \ln\left(\frac{5000}{3}\right)$

$t = \frac{\ln\left(\frac{5000}{3}\right)}{\ln(\sqrt{5})} \approx \boxed{9.2 \text{ hours}}$

6 The table (in text) gives U.S. pop, in millions for 1900-2000.

a Use (1900, 76), (1910, 92) to predict 2000 pop

t = yrs after 1900 $\boxed{N(t) = 76e^{kt}}$

$N(10) = 92 \Longrightarrow 76e^{10k} = 92$

$e^{10k} = \frac{92}{76} = \frac{46}{38} = \frac{23}{19}$

202 § 7.5 #s 6abc, 8abc, 11, 13ab, 18ab

6 a cnt'd

$e^{10K} = \frac{23}{19}$

$10K = \ln\left(\frac{23}{19}\right)$

$\boxed{K = \frac{1}{10}\ln\left(\frac{23}{19}\right)}$

$\therefore N(100) = 76\, e^{\frac{1}{10}\ln\left(\frac{23}{19}\right)(100)}$

$= 76\, e^{\ln\left(\frac{23}{19}\right)\left(\frac{1}{10}\right)(100)} = 76\left(e^{\ln\left(\frac{23}{19}\right)}\right)^{10}$

$= 76\left(\frac{23}{19}\right)^{10} \approx 513.5$ million!

This is much bigger!
Maybe a lot of immigration in 1900's?

b Use 1980 & 1990 figures to predict 2000 figure.

t = years after 1980:

$(0, 227), (10, 250)$

$N(t) = 227e^{kt}$

$N(10) = 227e^{10K} = 250$

$e^{10K} = \frac{250}{227}$

$10K = \ln\left(\frac{250}{227}\right)$

$K = \frac{1}{10}\ln\left(\frac{250}{227}\right)$

$N(20)$ = Pop in 2000 prediction

$= 227e^{\frac{1}{10}\ln\left(\frac{250}{227}\right)(20)}$

$= 227\left(\frac{250}{227}\right)^2 \approx$

≈ 275.3 million!
This is very close to 275

2010: $t = 30$:

$N(30) = 227e^{\frac{1}{10}\ln\left(\frac{250}{227}\right)(30)}$

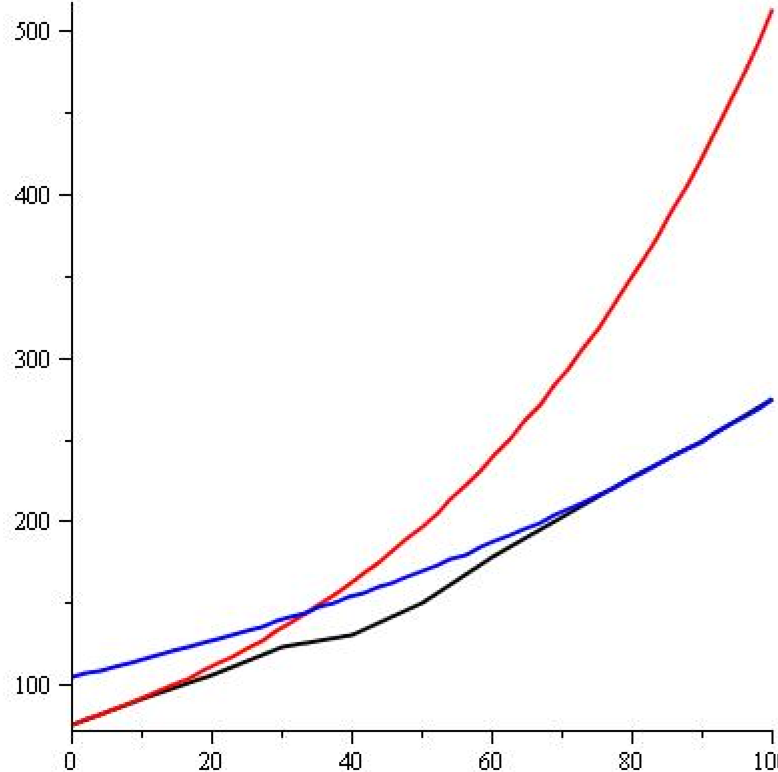
$= 227\left(\frac{250}{227}\right)^3 \approx \boxed{304 \text{ million}}$

2020: $t = 40$

$N(40) = 227\left(\frac{250}{227}\right)^4$

≈ 334 million in 2020

c See MAPLE Sketches, Next page.



Model using 1900 and 1910 data in red - It's the one that starts at the actual, but grows like crazy.
Model using 1980 and 1990 in blue - It's the one that starts high, but ends up close to actual at the end.
Model using actual connect-the-dots is in black. - It's the jagged one.

202 §7.5 #'s 8abc, 11, 13ab, 18ab

8) Bismuth-210 has $\frac{1}{2}$-life of 5 days.

(a) A sample has mass 800 mg. How much mass after t days? t = time, in days

$N(t) = 800e^{kt}$

$N(5) = 800e^{5k} = 400 = \frac{1}{2}$ what it started with.

$\Rightarrow e^{5k} = \frac{1}{2}$

$\Rightarrow 5k = \ln(\frac{1}{2})$

$\Rightarrow k = \frac{1}{5}\ln(\frac{1}{2})$

$$\boxed{N(t) = 800e^{\frac{1}{5}\ln(\frac{1}{2})t}}$$

OR $800(\frac{1}{2})^{\frac{1}{5}t}$

(b) When is the mass down to 1 mg?

$N(t) = 1$

$800(\frac{1}{2})^{\frac{1}{5}t} = 1$

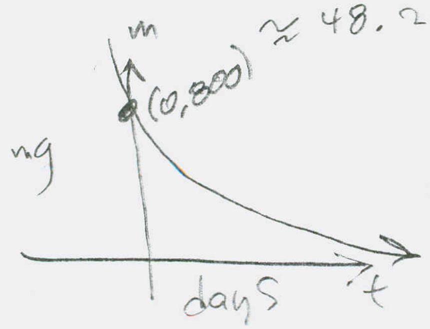
$(\frac{1}{2})^{\frac{1}{5}t} = \frac{1}{800}$

$\frac{1}{5}t = \log_{\frac{1}{2}}(\frac{1}{800})$

$t = 5\log_{\frac{1}{2}}(\frac{1}{800}) = \frac{5\ln(\frac{1}{800})}{\ln(\frac{1}{2})} = \frac{5\ln(800)}{\ln(2)}$

$\approx 48.2192809\!5 \approx \boxed{48 \text{ days}}$

(c)



202 §7.5 #s 11, 13ab, 18ab

(11) A parchment has 74% as much ^{14}C as plant matter of today. How old is it?

$\frac{1}{2}$ is 5730 yrs

$$N(t) = N_0 e^{kt}$$

$$N(5730) = \frac{1}{2} N_0$$

$$N_0 e^{5730k} = \frac{1}{2} N_0$$

$$e^{5730k} = \frac{1}{2}$$

$$5730k = \ln\left(\frac{1}{2}\right)$$

$$k = \frac{1}{5730} \ln\left(\frac{1}{2}\right)$$

$\therefore$ $N(t) = .74 N_0$;

$$N_0 e^{\frac{1}{5730}\ln(\frac{1}{2})t} = .74 N_0$$

$$\frac{1}{5730} \ln\left(\frac{1}{2}\right) t = \ln(.74)$$

$$t = 5730 \cdot \frac{\ln(.74)}{\ln(.5)}$$

$$\approx \boxed{2489 \text{ yrs old}}$$

$$N(t) = N_0 \left(\frac{1}{2}\right)^{kt}$$

$$N(5730) = \frac{1}{2} N_0$$

$$(N_0)\left(\frac{1}{2}\right)^{5730k} = \frac{1}{2} N_0$$

$$\left(\frac{1}{2}\right)^{5730k} = \frac{1}{2}$$

$$k = \frac{1}{5730}$$

$$\therefore N(t) = N_0 \left(\frac{1}{2}\right)^{\frac{1}{5730}t}$$

$$N_0 \left(\frac{1}{2}\right)^{\frac{1}{5730}t} = .74 N_0$$

$$\left(\frac{1}{2}\right)^{\frac{1}{5730}t} = .74$$

$$\frac{1}{5730} t = \log_{\frac{1}{2}}(.74)$$

$$t = 5730 \log_{\frac{1}{2}}(.74)$$

$$= \frac{5730 \ln(.74)}{\ln\left(\frac{1}{2}\right)} \checkmark$$

202 S'7.5 #s 13ab, 18ab

13 $T_s = 75°F$

T = temp in degrees F. $= T(t) = T$ as function of t = time (in min)

$T_0 = 185$ = initial temp

Let $y = T - T_s$

Then $\dfrac{dT}{dt} = \dfrac{dy}{dt}$

y = difference between Turkey temperature & Room temperature

$T(30) = 150$

$T(0) = 185$

$y(30) = 75$

$y(0) = 110$

$y = 110e^{kt}$

$y(30) = 110e^{30k} = 75$

$30k = \ln\left(\frac{75}{110}\right)$

$k = \frac{1}{30}\ln\left(\frac{75}{110}\right) = \frac{1}{30}\ln\left(\frac{15}{22}\right)$

a We find $y(45) = 110e^{45k} \approx 61.93 \approx 62°$

So $T = 62 + 75 = \boxed{137°F \text{ after } 45 \text{ minutes}}$

b We solve $y(t) = 25$ to find when temp will drop to $100°$:

$110e^{kt} = 25$

$e^{kt} = \frac{25}{110} = \frac{5}{22}$

$kt = \ln\left(\frac{5}{22}\right)$

$t = \frac{1}{k}\ln\left(\frac{5}{22}\right) = \dfrac{30\ln\left(\frac{5}{22}\right)}{\ln\left(\frac{15}{22}\right)}$

$\boxed{\approx 116 \text{ minutes}}$

202 §7.5 #s 18ab

18 (a) $1000 is borrowed @ 8% compounded

(i) Annually. After 3 years, we have

$$A = P\left(1+\frac{r}{m}\right)^{mt} = P(1+r)^t = 1000(1+.08)^3 \approx \boxed{\$1259.71} \quad m=1$$

(ii) quarterly. In 3 yrs,

$$A = 1000\left(1+\frac{.08}{4}\right)^{4\cdot 3} \approx \boxed{\$1268.24} \quad m=4$$

(iii) monthly, in 3 years,

$$A(3) = 1000\left(1+\frac{.08}{12}\right)^{12\cdot 3} \approx \boxed{\$1270.24} \quad m=12$$

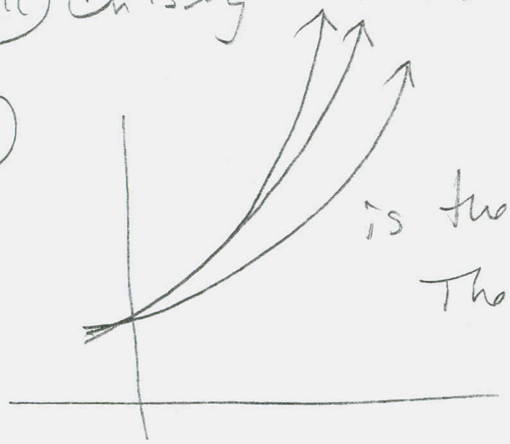
(iv) weekly $\Rightarrow A(3) = 1000\left(1+\frac{.08}{52}\right)^{52\cdot 3} \approx \boxed{\$1271.01}$ $m=52$

(v) daily $\Rightarrow A(3) = 1000\left(1+\frac{.08}{365}\right)^{365\cdot 3} \approx \boxed{\$1271.22}$ $m=365$

(vi) hourly $\Rightarrow A(3) = 1000\left(1+\frac{.08}{365\cdot 24}\right)^{365\cdot 24\cdot 3} \approx \boxed{\$1271.25}$ $m=365\cdot 24$

(vii) cntsly $\Rightarrow A(3) = 1000e^{.08\cdot 3} \approx \boxed{\$1271.25}$ $m=\infty$

(b)



is the general idea.
The taller function represents
the greater interest rate.