202 § 7.4 #s 3, 6, 18, 20, 26, 27, 33, 37, 42, 47, 63, 70, 77, 81

#s 1–26 Differentiate:

3) $f(x) = \log_2(1-3x)$

using change-of-base. $= \frac{1}{\ln(2)} \ln(1-3x)$

$\Rightarrow \boxed{f'(x) = \frac{1}{\ln(2)} \cdot \frac{-3}{1-3x}}$

Using Rote memory:

$\boxed{f'(x) = \frac{1}{\ln(2)} \cdot \frac{-3}{1-3x}}$

I never remember this. Always go back to Change-of-base.

6) $f(x) = \log_5(xe^x)$

$\Rightarrow f'(x) = \frac{1}{\ln(5)} \cdot \frac{e^x + xe^x}{xe^x}$

$\boxed{= \frac{1}{\ln(5)} \cdot \frac{x+1}{x}}$

18) $y = 10^{\tan(\theta)}$

using $\ln x$ & e^x are inverses

$= \left(e^{\ln(10)}\right)^{\tan(\theta)}$

$= e^{\ln(10)\tan\theta}$

$\Rightarrow y' = e^{\ln(10)\tan\theta} \cdot \ln(10)\sec^2(\theta)$

$= \left(e^{\ln(10)}\right)^{\tan\theta} \cdot \ln(10)\sec^2(\theta)$

$= 10^{\tan\theta} \cdot \ln(10)\sec^2(\theta)$

$y' = \ln(10) \cdot 10^{\tan(\theta)} \cdot \sec^2(\theta)$

by formula & Chain Rule.

Takes longer, but I can remember $\frac{d}{dx}[e^x] = e^x$ and the inverse property giving $b^x = e^{\ln(b)x}$

202 §7.4 #s 20, 26, 27, 33, 37, 42, 47, 63, 70, 77, 81

(20) $H(z) = \ln\sqrt{\dfrac{a^2-z^2}{a^2+z^2}} = \frac{1}{2}\ln\left(\dfrac{a^2-z^2}{a^2+z^2}\right)$

$= \frac{1}{2}\left[\ln(a^2-z^2) - \ln(a^2+z^2)\right] \longrightarrow$

$$\boxed{H'(z) = \frac{1}{2}\left[\frac{-2z}{a^2-z^2}\right] - \frac{1}{2}\left[\frac{2z}{a^2+z^2}\right]}$$

(26) $y = 2^{3^{x^2}}$

$\ln(y) = \ln\left(2^{3^{x^2}}\right) = \ln(2)\cdot 3^{x^2}$

$\dfrac{y'}{y} = \ln(2)\ln(3)(2x)\cdot 3^{x^2}$

$$\boxed{y' = 2\ln(2)\ln(3)\,x\cdot 3^{x^2}\cdot 2^{3^{x^2}}}$$

(27) Find y' & y'' for $y = x^2\ln(2x) \longrightarrow$

$y' = 2x\ln(2x) + x^2\cdot\dfrac{2}{2x} = \boxed{2x\ln(2x) + x} = y'$

$y'' = 2\ln(2x) + 2x\cdot\dfrac{2}{2x} + 1 = \boxed{2\ln(2x) + 2 = y''}$

(33) Find $f'(x)$ & $\mathcal{D}(f)$ for $f(x) = \ln(x^2-2x)$

$$\boxed{\begin{array}{l}\mathcal{D}(f) = (-\infty,0)\cup(2,\infty)\\ f'(x) = \dfrac{2x-2}{x^2-2x}\end{array}}$$

202 §7.4 #s 37, 42, 47, 63, 70, 77, 81

(37) Find an eq'n of the tangent line to $y = \ln(xe^{x^2})$ @ $(1,1)$:

$$y' = \frac{e^{x^2} + x \cdot 2xe^{x^2}}{xe^{x^2}} = \frac{1+2x^2}{x}$$

$$\boxed{y = 3(x-1)+1}$$

$$= 3x - 2 = y$$

$$y'(1) = \frac{1+2}{1} = 3 = m$$

~~42~~ #s 41-52 Use logarithmic differentiation to find the derivative.

(42) $y = \sqrt{x}\ e^{x^2}(x^2+1)^{10}$

$$\ln(y) = \ln\left(x^{\frac{1}{2}} e^{x^2} (x^2+1)^{10}\right)$$

$$= \frac{1}{2}\ln(x) + x^2 + 10\ln(x^2+1) \longrightarrow$$

$$\frac{y'}{y} = \frac{1}{2}\cdot\frac{1}{x} + 2x + 10\cdot\frac{2x}{x^2+1}$$

$$\longrightarrow \boxed{y' = \left(\frac{1}{2x} + 2x + \frac{20x}{x^2+1}\right)\sqrt{x}\ e^{x^2}(x^2+1)^{10}}$$

(47) $y = x^{\sin(x)}$

$$\ln(y) = (\sin(x))\ln(x)$$

$$\frac{y'}{y} = (\cos(x))\ln(x) + (\sin(x))\left(\frac{1}{x}\right)$$

$$\boxed{y' = \left(\cos(x)\ln(x) + \frac{\sin(x)}{x}\right)x^{\sin(x)}}$$

202 §7.4 #s 63, 70, 77, 81

(63) Graph the curve using §4.5 stuff.

$f(x) = \ln(x^2+1)$

(A) $\mathcal{D} = \mathbb{R}$, since $x^2+1 \geq 1 > 0$.

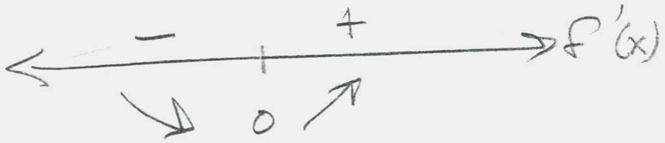
(B) $f(0) = \ln(1) = 0 \rightsquigarrow (0,0)$

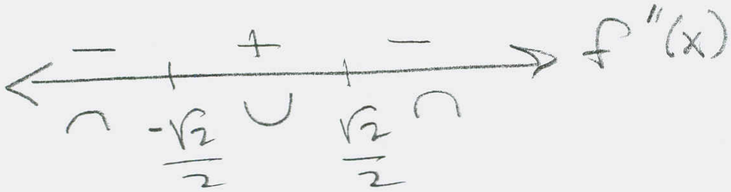
(C) $\ln((-x)^2+1) = \ln(x^2+1) \Longrightarrow$ even $\Longrightarrow$ symmetric about the y-axis

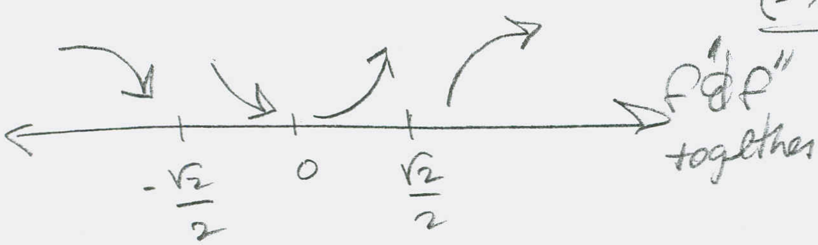
(D) Asymptotes: NONE

$\boxed{E}$ $f'(x) = \dfrac{2x}{x^2+1}$

critical @ $x = 0$







$\ln\left(\left(\frac{\sqrt{2}}{2}\right)^2+1\right) = \ln\left(\frac{1}{2}+1\right) = \ln\left(\frac{3}{2}\right)$

$$f''(x) = \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2}$$

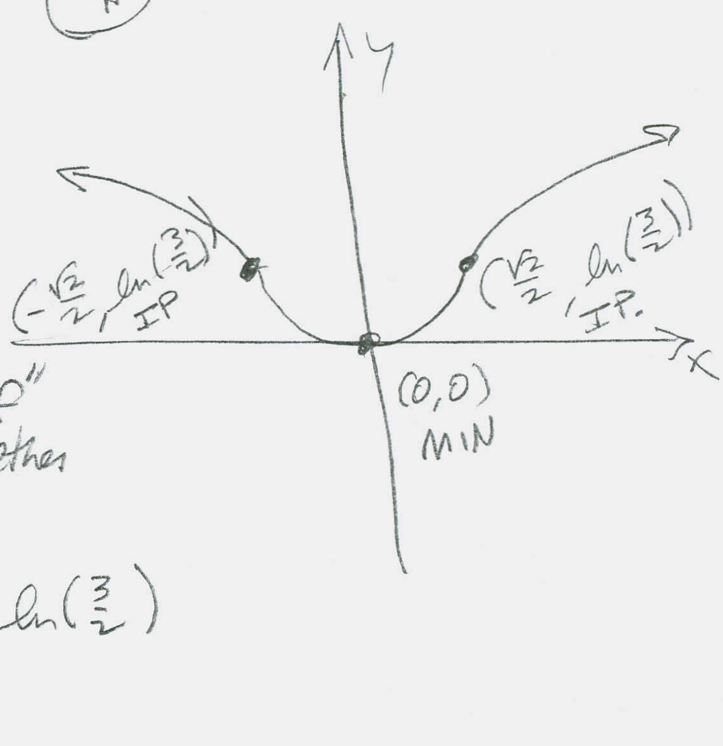
$$= \frac{2x^2+2-4x^2}{(x^2+1)^2}$$

(F), (G)

$$= \frac{-2(2x^2-1)}{(x^2+1)^2} \overset{\text{SET}}{=} 0$$

$$\Longrightarrow x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

(H)



202 § 7.4 #s 70, 77, 81

#s 69–80 evaluate the integral.

(70) $\int_1^2 \frac{u^2+4}{u^3}\,du = \int_1^2 \frac{du}{u} + \int_1^2 u^{-3}\,du$

$= \ln|u|\Big]_1^2 - \frac{1}{2}u^{-2}\Big]_1^2 = \ln(2) - \ln(1) - \left[\frac{1}{2}\cdot\frac{1}{2^2} - \frac{1}{2}\cdot\frac{1}{1^2}\right]$

$= \ln(2) - \frac{1}{8} + \frac{1}{2} = \boxed{\ln(2) + \frac{3}{8}}$

(77) $\int \frac{\sin(2x)}{1+\cos^2 x}\,dx = \int \frac{\sin(2x)}{1 + \left(\frac{1+\cos(2x)}{2}\right)}\,dx$

$= \int \frac{\sin(2x)}{\frac{3+\cos(2x)}{2}}\,dx = 2\int \frac{\sin(2x)}{3+\cos(2x)}\,dx = -\int \frac{-2\sin(2x)\,dx}{3+\cos(2x)}$

$= -\int \frac{du}{u} = -\ln|u| + C = \boxed{-\ln|3+\cos(2x)| + C}$

(81) Claim: $\int \cot(x)\,dx = \ln|\sin x| + C$ (a) Diff. RHS. (b) Example 11

(a) $\mathcal{D}(\cot x) = \{x \mid x \neq n\pi, n \in \mathbb{Z}\}$ Let's use

$x \in (0,\pi) \Rightarrow \sin(x) > 0 \Rightarrow \ln|\sin x| = \ln(\sin x)$

$\frac{d}{dx}\left[\ln|\sin x| + C\right] = \frac{\cos x}{\sin x} = \cot x$ ■

(b) $\int \frac{\cos(x)}{\sin(x)}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln|\sin x| + C$ ■