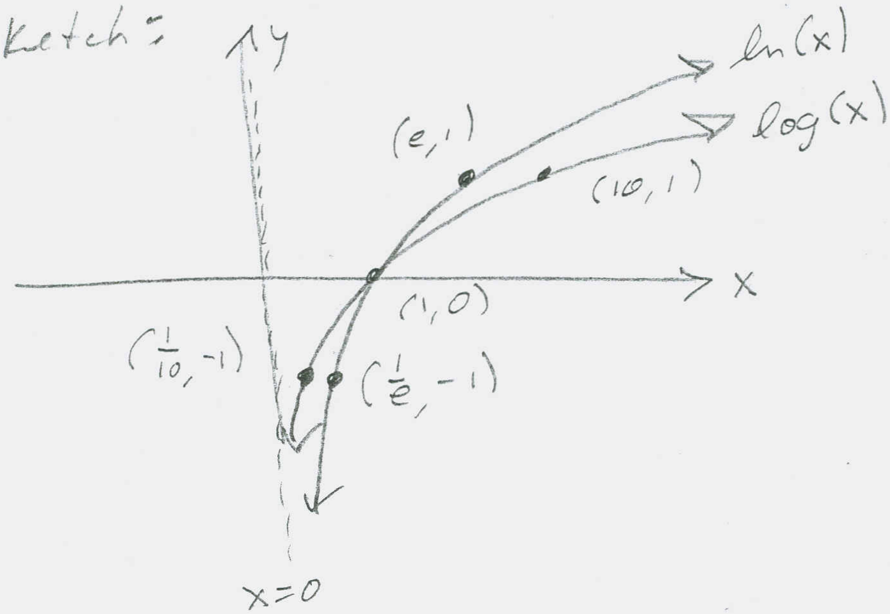
202 §7.3 #s 2abc, 4ab, 7ab, 10, 18, 22, 23, 26, 28, 33, 36, 50, 54, 60, 62, 65

(2) (a) The natural logarithm is log to the base e, $\ln(x) = \log_e(x)$. Its output is the power of e that equals x. That is, if $x = e^y$, then $\ln(x) = y$.

(b) Same as before, only the base is 10, when we're discussing common logarithms.

(c) Sketch:



(4) Find the exact value of each expression:

(a) $\ln\left(\frac{1}{e}\right)$

$= \ln(e^{-1}) =$

$= \boxed{-1}$

(b) $\log(\sqrt{10})$

$= \log\left(10^{\frac{1}{2}}\right)$

$= \boxed{\frac{1}{2}}$

202 §7.3 #s 7ab, 10, 18, 22, 23, 26, 28, 33, 36, 50, 54, 60, 62, 65

(7) Same as #4

(a) $\log_2(6) - \log_2(15) + \log_2(20)$

$= \log_2\left(\frac{\cancel{6}^{2} \cdot \cancel{20}^{4}}{\cancel{15}_{\cancel{5}}}\right) = \log_2(8) = \log_2(2^3) = \boxed{3}$

(b) $\log_3(100) - \log_3(18) - \log_3(50)$

$= \log_3\left(\frac{100}{18 \cdot 50}\right) = \log_3\left(\frac{1}{9}\right)$ $\frac{\cancel{100}^{\cancel{2}^{1}}}{\cancel{18}_{9} \cdot \cancel{50}_{1}}$

$= \log_3(3^{-2}) = \boxed{-2}$

(10) Use the properties of logarithms to expand:

$\ln\left(\sqrt{a(b^2+c^2)}\right) = \boxed{\frac{1}{2}\ln(a) + \frac{1}{2}\ln(b^2+c^2)}$

(18) Express as a single logarithm

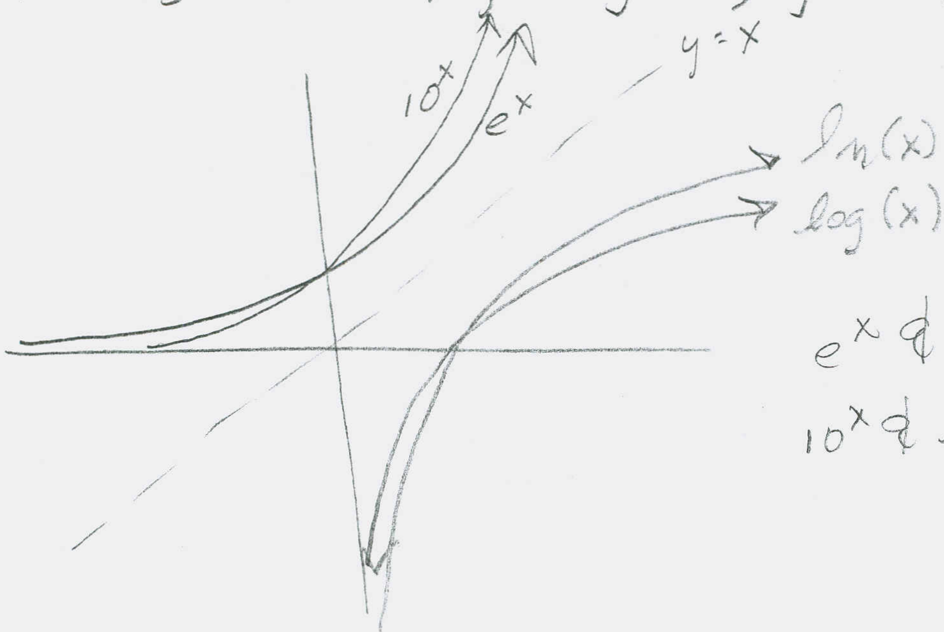
$\ln(a+b) + \ln(a-b) - 2\ln(c)$

$= \boxed{\ln\left(\frac{(a+b)(a-b)}{c^2}\right)}$

(22) Use Formula 7 to graph the given functions on a common screen. How are these graphs related?

202 §7.3 #s 22, 23, 26, 28, 33, 36, 50, 54, 60, 62, 65

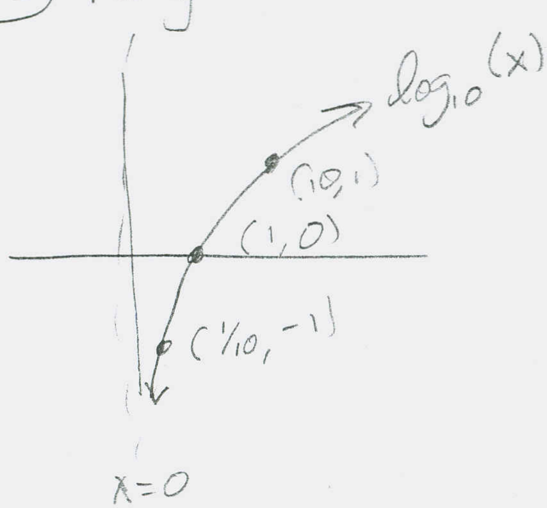
(22) $y = \ln(x)$, $y = \log(x)$, $y = e^x$, $y = 10^x$

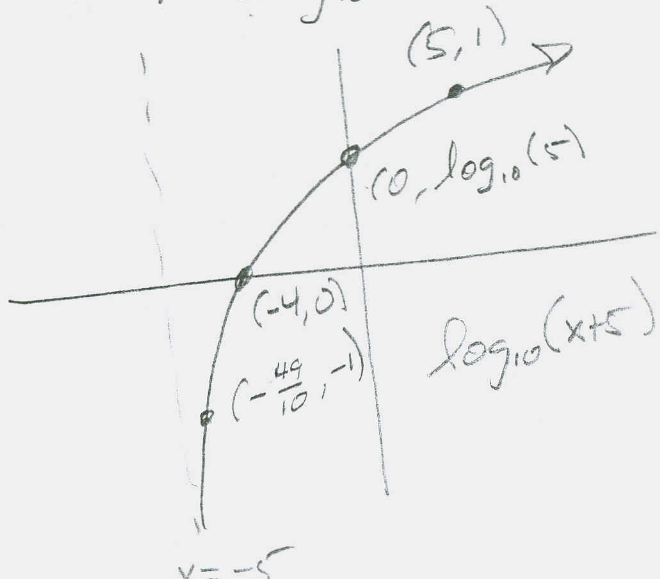


e^x & $\ln x$ are inverses

10^x & $\log x$ are inverses

(23) Rough sketch of $y = f(x) = \log_{10}(x+5)$





#s 25–34 Solve each eq'n for x.

(26) $e^{2x+3} - 7 = 0$

$e^{2x+3} = 7$

$2x+3 = \ln(7)$

$2x = \ln(7) - 3$

$\boxed{x = \frac{1}{2}\ln(7) - \frac{3}{2}}$

202 § 7.3 #s 28, 33, 36, 50, 54, 60, 62, 65

28 $e^{3x+1} = k$

$3x+1 = \ln(k)$

$3x = \ln(k) - 1$

$$\boxed{x = \frac{\ln(k)-1}{3}}$$

33 $e^{2x} - e^{x} - 6 = 0$

Let $u = e^x$. Then

$u^2 - u - 6 = 0$

$(u-3)(u+2) = 0$

$u = 3$ OR $u = -2$ NO

$e^{2x} > 0$!

$e^{2x} = 3$

$2x = \ln(3)$

$$\boxed{x = \frac{1}{2}\ln(3)}$$

36 Solve each equation to nearest 4th decimal place.

a $\ln(1+\sqrt{x}) = 2$

$1 + \sqrt{x} = e^2$

$\sqrt{x} = e^2 - 1$

$x = (e^2-1)^2 \approx$

b $3^{\frac{1}{x-4}} = 7$

$\frac{1}{x-4} = \log_3(7)$

$x - 4 = \frac{1}{\log_3(7)}$

$x = \frac{1}{\log_3(7)} + 4$

$= \frac{1}{\frac{\ln(7)}{\ln(3)}} - 4$

$= \frac{\ln(3)}{\ln(7)} - 4 \approx$

202 § 7.3 #s 50, 54, 60, 62, 65

(50) Find the limits

$$\lim_{x\to\infty}\left[\ln(x+2)-\ln(x+1)\right]=\lim_{x\to\infty}\left[\ln\left(\frac{x+2}{x+1}\right)\right]$$

$$=\ln\left[\lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)\right]=\ln(1)=0.$$

Moving the limit inside a function is legit, provided that function behaves itself.

(54) Find $\mathcal{D}(f)$ & Find f^{-1} & $\mathcal{D}(f^{-1})$

$f(x)=\ln(\ln x+2)$

$\mathcal{D}$: Need $\ln x+2>0$

$\ln x > -2$

$x > e^{-2}$

$$\boxed{\begin{gathered}\mathcal{D}=\{x \mid x>e^{-2}\}=(e^{-2},\infty)\\ R=(-\infty,\infty)\end{gathered}}$$

Because the range of $\ln(x)+2$, restricted to $x\in\mathcal{D}$, covers all of $(0,\infty)$, hence $\ln(\ln x+2)$ covers the entire range of the $\ln(x)$ function.

$x=\ln(\ln(y)+2)=x$

$\ln(y)+2=e^{x}$

$\ln(y)=e^{x}-2$

$y=\boxed{e^{e^{x}-2}=f^{-1}(x)}$

$$\boxed{\begin{gathered}\mathcal{D}=(-\infty,\infty)\\ R=(e^{-2},\infty)\end{gathered}}$$

Your intuition will grow, as you see more of these.

202 §7.3 #s 60, 62, 65

(60) Find $f^{-1}(x)$ for $f(x) = \dfrac{e^x}{2e^x+1}$

$$\frac{e^y}{2e^y+1} = x$$

$$e^y = x(2e^y+1)$$

$$e^y = 2xe^y + x$$

$$e^y - 2xe^y = x$$

$$e^y(1-2x) = x$$

$$e^y = \frac{x}{1-2x}$$

$$y = \ln\left(\frac{x}{1-2x}\right)$$

(62) On what interval is $f(x) = 2e^x - e^{-3x}$ concave downward?

$\mathcal{D} = \mathbb{R}$

$$f'(x) = 2e^x + 3e^{-3x}$$

$$f''(x) = 2e^x - 9e^{-3x}$$

$$= e^{-3x}\left[2e^{4x} - 9\right] \overset{\text{SET}}{=} 0$$

$$2e^{4x} = 9$$

$$e^{4x} = \frac{9}{2}$$

$$4x = \ln\left(\frac{9}{2}\right)$$

$$x = \frac{1}{4}\ln\left(\frac{9}{2}\right)$$

$$\overset{-}{\longleftarrow\!\!-\!\!-\!\!-}\underset{\frac{1}{4}\ln\left(\frac{9}{2}\right)}{|}\overset{+}{-\!\!-\!\!-\!\!\longrightarrow}\ f''$$

Concave down on $\left(-\infty, \frac{1}{4}\ln\left(\frac{9}{2}\right)\right)$

(65) (a) Show that $f(x) = \ln(x + \sqrt{x^2+1})$ is odd. DONE IN CLASS.

(b) Find its inverse.

(a) $f(-x) = \ln(-x + \sqrt{x^2+1})$

$$= \ln\left(\left(\sqrt{x^2+1} - x\right)\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}\right)\right)$$

$$= \ln\left(\frac{1}{\sqrt{x^2+1}+x}\right) = -\ln\left(\sqrt{x^2+1}+x\right)$$

$$= -f(x) \implies \text{ODD.}$$

(b) $\cancel{\ln(y + \sqrt{y^2+1}) = x}$

$\cancel{y + \sqrt{y^2+1} = e^x}$

202 §7.3 #65

65b

$$\ln\left(y+\sqrt{y^2+1}\right)=x$$

$$y+\sqrt{y^2+1}=e^x$$

$$\sqrt{y^2+1}=e^x-y$$

$$y^2+1=e^{2x}-2ye^x+y^2$$

$$2ye^x=e^{2x}-1$$

$$\boxed{y=\frac{e^{2x}-1}{2e^x}}$$