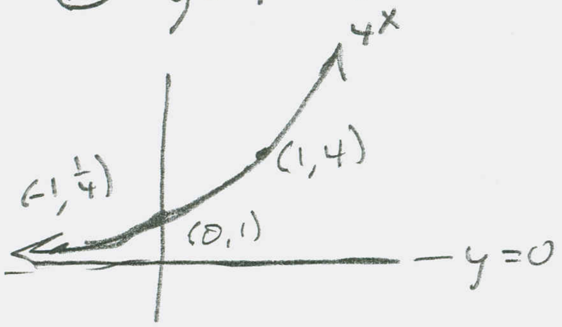
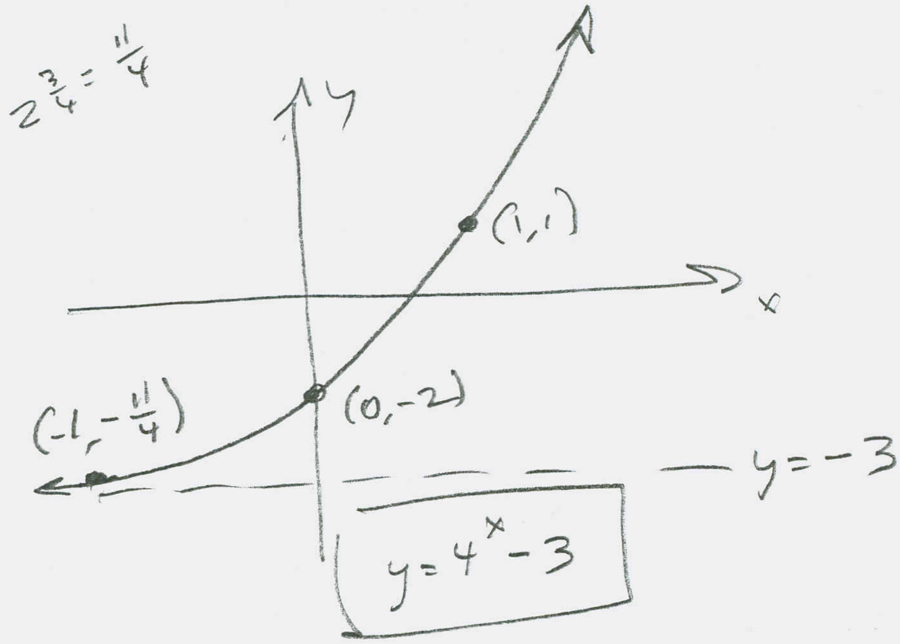
202 §7.2 #s 7, 11, 13, 16ab, 31, 34, 38, 44, 47, 54, 72, 73, 74, 80, 84

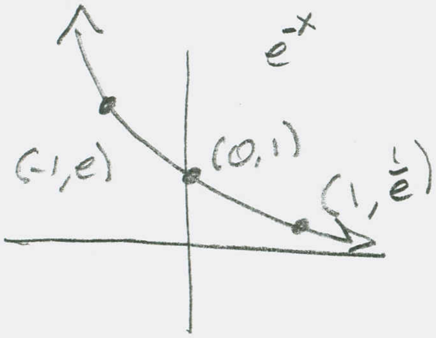
#s 7–12 Sketch

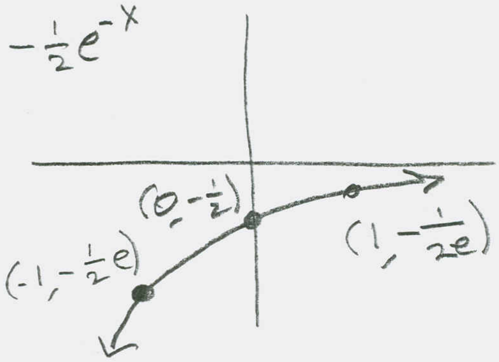
(7) $y = 4^x - 3$

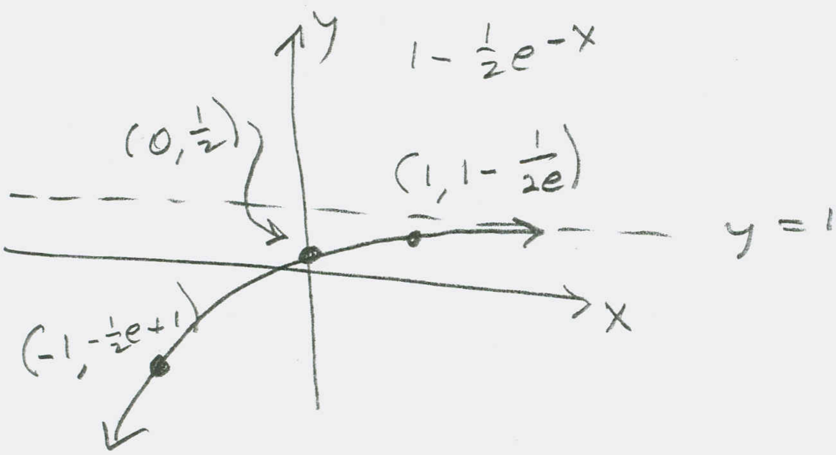




(11) $y = 1 - \frac{1}{2}e^{-x}$







202 § 7.2 #s 13, 16ab, 31, 34, 38, 44, 47, 54, 72, 73, 74, 80, 84

13 Start w/ e^x. Find function corresponding to transformation

(a) 2 units down : $\boxed{f(x)-2 = e^x - 2}$

(b) 2 units right : $\boxed{f(x-2) = e^{x-2}}$

(c) reflect about x-axis : $\boxed{-f(x) = -e^x}$

(d) reflect " y-axis : $\boxed{f(-x) = e^{-x}}$

(e) " " x-axis and then y-axis

: $f(x) \longrightarrow -f(x) \longrightarrow \boxed{-f(-x) = -e^{-x}}$

16 Find the $\mathcal{D}$ of...

(a) $g(t) = \sin(e^{-t})$

$\mathcal{D} = \mathbb{R}$, since $\mathcal{D}(e^{-t})$ is $\mathbb{R}$ and so is $\mathcal{D}(\text{sine})$

(b) $g(t) = \sqrt{1-2^t}$

Need $1-2^t \geq 0$

$\Rightarrow -2^t \geq -1$

$\Rightarrow 2^t \leq 1$

$\Rightarrow t \leq 0$, since 2^t is increasing and $2^t = 1$ when $t = 0$.

$$\boxed{\mathcal{D} = \{t \mid t \leq 0\} \text{ OR } (-\infty, 0]}$$

202 §7.2 #s 31, 34, 38, 44, 47, 54, 72-74, 80, 84

(31) Differentiate $f(x) = (x^3+2x)e^x$

$f'(x) = (3x^2+2)e^x + (x^3+2x)e^x$ is fine

$= (x^3+3x^2+2x+2)e^x$ is pretty

oops! Same instructions thru #46

(34) $y = e^u(\cos(u) + cu) \Rightarrow \boxed{y' = e^u(\cos(u)+cu) + e^u(-\sin(u) + c)}$

(38) $f(t) = \sin(e^t) + e^{\sin(t)}$

$\Rightarrow f'(t) = \cos(e^t)\cdot e^t + e^{\sin(t)} \cdot \cos(t)$

(44) $y = \sqrt{1 + xe^{-2x}} \Rightarrow \boxed{y' = \frac{1}{2}(1+xe^{-2x})^{-\frac{1}{2}}(e^{-2x} - 2xe^{-2x})}$

(47) Find eq'n of tangent line to $f(x) = e^{2x}\cos(\pi x)$ @ $(0,1)$:

$f'(x) = 2e^{2x}\cos(\pi x) - \pi e^{2x}\sin(\pi x) \Rightarrow$

$f'(0) = 2\cdot 1 \cdot 1 - \pi \cdot 1 \cdot 0 = 2 = m_{tan} \Rightarrow$

$\boxed{y = 2(x-0)+1}$

202 §7.2 #s 54, 72-74, 80, 84

(54) Find $\lambda \ni y = e^{\lambda x}$ satisfies $y + y' = y''$

$\Rightarrow y + Dy = D^2 y \Rightarrow D^2 y - Dy - y = 0$

$\Rightarrow (D^2 - D - 1)y = 0 \Rightarrow y = 0$ (No help) or

$D^2 - D - 1 = 0 \Longrightarrow$

$D^2 - D + \left(\frac{1}{2}\right)^2 = 1 + \frac{1}{4}$

$\left(D - \frac{1}{2}\right)^2 = \frac{5}{4}$

$D - \frac{1}{2} = \pm \frac{\sqrt{5}}{2}$

$D = \boxed{\frac{1 \pm \sqrt{5}}{2} = \lambda}$

(72) $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ is normal density curve.

μ = mean, σ = standard deviation.

Let's re-scale & use $\mu = 0$, for simplicity:

$f(x) = e^{-\frac{x^2}{2\sigma^2}}$

(a) Find Asymptote, max, inflection pts.

$\boxed{y = 0 \text{ is H.A.}}$

$f'(x) = -\frac{2x}{2\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \overset{\text{SET}}{=} 0 \Rightarrow x = 0$

$f(x)$ increasing on $(-\infty, 0)$, decreasing on $(0, \infty)$

$\boxed{\text{So, } (0,1) \text{ is max pt.}}$

202 §7.2 #s 72–74, 80, 84

72 cnt'd

(a) cnt'd $f''(x) = \frac{d}{dx}\left[-\frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}\right]$

$= -\frac{1}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} + \frac{x}{\sigma^2} \cdot \frac{2x}{2\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$

$= -\frac{1}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} + \frac{2x^2}{2\sigma^4} e^{-\frac{x^2}{2\sigma^2}}$

$= \frac{1}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \left[-1 + \frac{x^2}{\sigma^2}\right] \overset{SET}{=} 0$

$\frac{x^2 - \sigma^2}{\sigma^2} = \frac{(x-\sigma)(x+\sigma)}{\sigma^2} = 0 \Longrightarrow \boxed{x = \pm\sigma}$

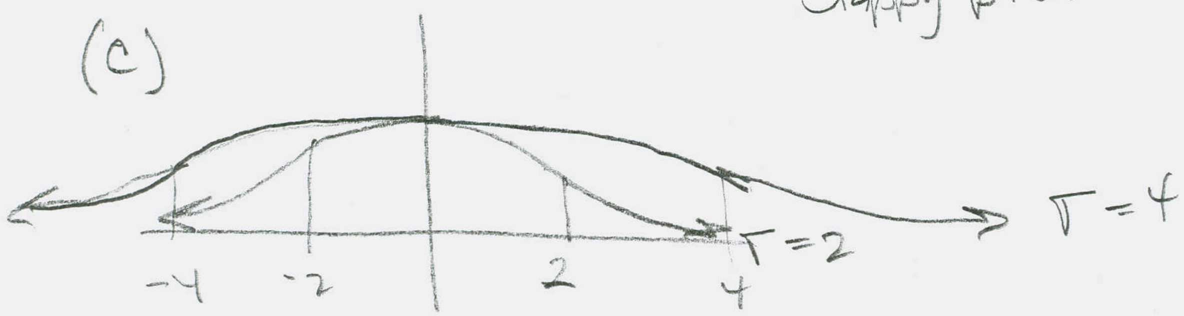
$f(\sigma) = e^{-\frac{\sigma^2}{2\sigma^2}} = e^{-\frac{1}{2}} = f(-\sigma)$

$\boxed{(\sigma, e^{-\frac{1}{2}}), (-\sigma, e^{-\frac{1}{2}}) \text{ are I.P.s}}$

(b) σ is distance from $x=0$ to I.P.'s x-coordinate.

Crappy pic.

(c)



202 §7.2 #s 73, 74, 80, 84

#s 73-82 Evaluate the integral

(73) $\int_0^5 e^{-3x}\,dx$ $\qquad u=-3x \Rightarrow du=-3dx$

$\Rightarrow dx=-\frac{1}{3}du$

$x=0 \Rightarrow u=0$

$x=5 \Rightarrow u=-15$

$= \int_0^{-15} e^u\left(-\frac{1}{3}du\right)$

$= -\frac{1}{3}\left[e^u\right]_0^{-15} = -\frac{1}{3}\left[e^{-15}-e^0\right] = -\frac{1}{3}\left[e^{-15}-1\right]$

$= \boxed{\frac{1}{3}-\frac{1}{3}e^{-15}}$

$u=-x^2 \Rightarrow du=-2x\,dx$

(74) $\int_0^1 xe^{-x^2}dx$ $\qquad \Rightarrow dx=\frac{du}{-2x}$

$x=0 \Rightarrow \cancel{x}\,u=0$

$x=1 \Rightarrow u=-1$

$= \int_0^{-1} x\,e^u\left(\frac{du}{-2x}\right)$

$= \int_0^{-1} e^u \cdot -\frac{1}{2}du$

$= -\frac{1}{2}\int_0^{-1} e^u du = -\frac{1}{2}\left[e^u\right]_0^{-1} = -\frac{1}{2}\left[e^{-1}-e^0\right]$

$= \boxed{\frac{1}{2}-\frac{1}{2e}}$

202 §7.2 #s 80, 84

80 $\int \frac{e^{\frac{1}{x}}}{x^2}\,dx$

$u = \frac{1}{x} \Rightarrow du = -\frac{1}{x^2}\,dx$

$\Rightarrow dx = -x^2\,du$

$= \int e^u \cdot \frac{1}{x^2} \cdot \left(-x^2\,du\right)$

$= -\int e^u\,du = -e^u + C = \boxed{-e^{\frac{1}{x}} + C}$

84 Find $f(x)$ if $f''(x) = 3e^x + 5\sin x$,

$f(0) = 1$ & $f'(0) = 2$

$f'(x) = 3e^x - 5\cos x + C$

$f'(0) = 3 - 5 + C = 2$

$\Rightarrow -2 + C = 2$

$\Rightarrow C = 4$

$\Rightarrow f'(x) = 3e^x - 5\cos x + 4$

$\Rightarrow f(x) = 3e^x - 5\sin x + 4x + D$

$\Rightarrow f(0) = 3 + 4 + D = 1$

$\Rightarrow 7 + D = 1$

$\Rightarrow D = -6 \Rightarrow \boxed{f(x) = 3e^x - 5\sin x + 4x - 6}$