202 S 7.1 #s 9, 12, 14, 15, 17, 18, 24, 28, 30, 32, 33, 36, 37, 40, 42

#s 3-16 Is it 1-to-1?

9 $f(x) = x^2 - 2x$

i

$x(x-2) = 0$

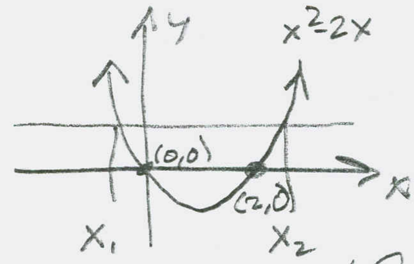
$x = 0,\ x = 2$

$f(0) = f(2) = 0 \Longrightarrow$

NO

Graphical

ii




Flunks horizontal line test. No

Algebraic

iii

$x_1^2 - 2x_1 = x_2^2 - 2x_2$

$x_1^2 - 2x_1 + 1 = x_2^2 - 2x_2 + 1$

$(x_1 - 1)^2 = (x_2 - 1)^2$

$|x_1 - 1| = |x_2 - 1|$

$x_1 - 1 = x_2 - 1$ OR $x_1 - 1 = -(x_2 - 1)$

$x_1 = x_2$ OR $x_1 - 1 = -x_2 + 1$

$x_1 = -x_2 + 2$

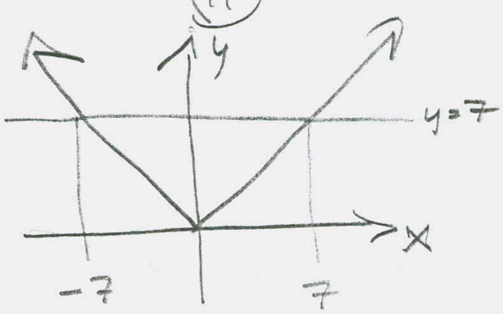
Two values of x give same value of y. No.

12 $g(x) = |x|$

i Degenerate

$x = 0$ OR $x = -0$ gives nada.

ii




Flunks HLT.

iii

$|x_1| = |x_2|$

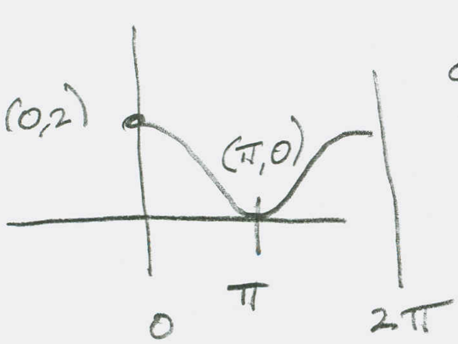
$x_1 = x_2$ OR $x_1 = -x_2$

Two possibilities. No.

14 $1 + \cos x = f(x)$ on $[0, \pi]$

DONE IN CLASS.

i Graph it




You see $f(x)$ is decreasing on $[0, \pi]$

ii Calculus:

$f'(x) = 1 - \sin x \overset{\text{SET}}{=} 0$

$\Longrightarrow \sin x = 1$

$\Longrightarrow x = 0,\ x = \pi$ and

$f'(x) < 0$ on $(0, \pi)$

$\Longrightarrow$ 1-to-1 on $[0, \pi]$.

202 §7.1 #s 15, 17, 18, 24, 28, 30, 32, 33, 36, 37, 40, 42

15 $f(t)$ is the height of a football $t$ sec after kickoff.

No. It hits same height going up & coming down.

17 If $f$ is 1-to-1 & $f(2) = 9$, then $\boxed{f^{-1}(9) = 2}$

18 $f(x) = x + \cos x$. We find $f^{-1}(1)$:

$f(x) = x + \cos x \overset{\text{SET}}{=} 1$

$\Rightarrow x = 0$ (by good guess / grapher / Newton's method)

$\Rightarrow \boxed{f^{-1}(1) = 0}$ (since $f(0) = 1$)

#s 24–28 Find $f^{-1}(x)$

24 $f(x) = \dfrac{4x-1}{2x+3} \Rightarrow$

$$x = \frac{4y-1}{2y+3}$$

$x(2y+3) = 4y - 1$

$2xy + 3x = 4y - 1$

$2xy - 4y = -3x - 1$

$(2x-4)y = -3x - 1$

$$y = \frac{-3x-1}{2x-4} = \boxed{-\frac{3x+1}{2x-4} = f^{-1}(x)}$$

202 § 7.1 #s 28, 30, 32, 33, 36, 37, 40, 42

(28) $f(x) = 2x^2 - 8x,\ x \geq 2$

$x = 2y^2 - 8y$

$\frac{1}{2}x = y^2 - 4y$

$y^2 - 4y = \frac{1}{2}x$

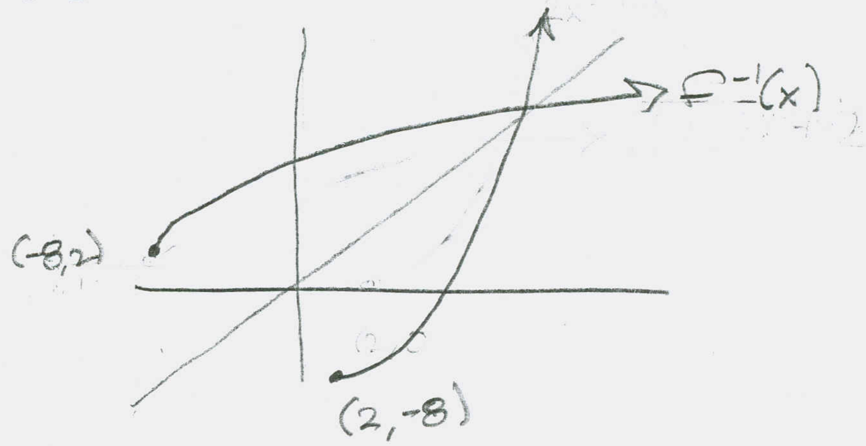
$y^2 - 4y + 2^2 = \frac{1}{2}x + 4$

$(y-2)^2 = \frac{1}{2}x + 4$

$y - 2 = \pm\sqrt{\frac{1}{2}x + 4} \longrightarrow$

$$\boxed{y = \sqrt{\tfrac{1}{2}x + 4} + 2 = f^{-1}(x)}$$

$\left.\begin{array}{l} \mathcal{D} = [-8, \infty) \\ \mathcal{R} = [2, \infty) \end{array}\right\}$ From f & variable swap.




$y = 2x^2 - 8x$

$= 2(x^2 - 4x + 2^2) - 8$

$= 2(x-2)^2 - 8$

$\mathcal{D} = [2, \infty)$ given

$\mathcal{R} = [-8, \infty)$ from graph

(#30) Find $f^{-1}$ & use it to graph $f$, $f^{-1}$ & $y = x$ on same screen. (ZOOM-SQUARE)

$f(x) = \sqrt{x^2 + 2x},\ x > 0$

$\sqrt{y^2 + 2y} = x$

$y^2 + 2y = x^2$

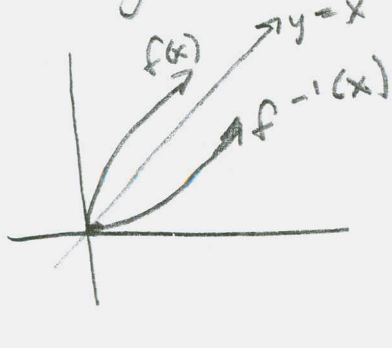
$y^2 + 2y + 1 = x^2 + 1$

$(y+1)^2 = x^2 + 1$

$y + 1 = \pm\sqrt{x^2 + 1} \longrightarrow$

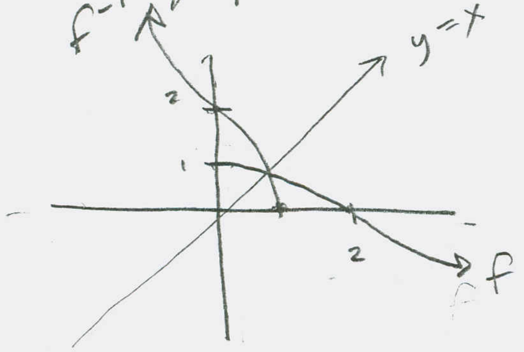
$$\boxed{f^{-1}(x) = y = \sqrt{x^2 + 1} - 1}$$

Use top 1/2 to match up with right half of $f(x)$



202 #5 ~~30~~, 32, 33, 36, 37, 40, 42

(32) Use graph of $f$ to graph $f^{-1}$




#s 33–36 (a) Show $f$ is 1-to-1

(b) Use $\boxed{T7}$ to find $(f^{-1})'(a)$

(c) Find $f^{-1}(x)$ & give its $\mathcal{D}$ & $\mathcal{R}$.

(d) Use (c) to check (b)

(e) Sketch $f$ & $f^{-1}$ on same axes.

(33) $f(x) = x^3,\ a = 8$

1-to-1 ✓

(a) $f'(x) = 3x^2 > 0\ \forall\, x \neq 0$ & @ $x = 0$, it's a terrace.

(b) $f^{-1}(8) = 2$, so $(f^{-1})'(8) = \dfrac{1}{f'(f^{-1}(8))} = \dfrac{1}{f'(2)}$

$= \dfrac{1}{3(2)^2} = \boxed{\dfrac{1}{12} = (f^{-1})'(8)}$

(c) $y^3 = x$

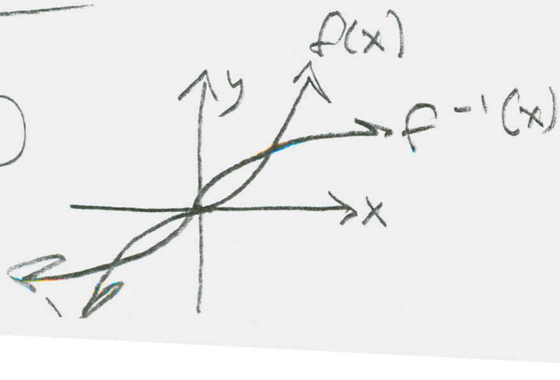
$\boxed{f^{-1}(x) = y = \sqrt[3]{x}}$ $\boxed{\mathcal{D} = \mathcal{R} = \mathbb{R}}$

(d) $(f^{-1})'(x) = \frac{1}{3}x^{-2/3}$

$(f^{-1})'(8) = \frac{1}{3}(8)^{-\frac{2}{3}} = \frac{1}{3}(2)^{-2}$

$= \left(\frac{1}{3}\right)\left(\frac{1}{2^2}\right) = \boxed{\frac{1}{12}}$ ✓

(e)



202 §7.1 #s 36, 37, 40, 42

(36) $f(x) = \frac{1}{x-1},\ x>1,\ a=2$  $\mathcal{D}=(1,\infty),\ \mathcal{R}=(0,\infty)$

$f^{-1}: \mathcal{D}=(0,\infty),\ \mathcal{R}=(1,\infty)$

(a) $\frac{1}{x_1-1} = \frac{1}{x_2-1}$

$x_2-1 = x_1-1$

$x_2 = x_1$ ✓

(b) $f'(x) = -(x-1)^{-2} = -\frac{1}{(x-1)^2}$

$f(x) \overset{\text{SET}}{=} 2 \Longrightarrow$

$\frac{1}{x-1} = 2$

$1 = 2x-2$

$2x-2=1$

$2x=3$

$x=\frac{3}{2} = f^{-1}(2)$

$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))}$

$= \frac{1}{f'(\frac{3}{2})} = \frac{1}{-\frac{1}{(\frac{3}{2}-1)^2}}$

$= -(\frac{3}{2}-1)^2$

$= -(\frac{1}{2})^2 = -\frac{1}{4}$

$\boxed{(f^{-1})'(2) = -\frac{1}{4}}$

(c) $\frac{1}{y-1} = x$

$1 = xy - x$

$xy - x = 1$

$xy = x+1$

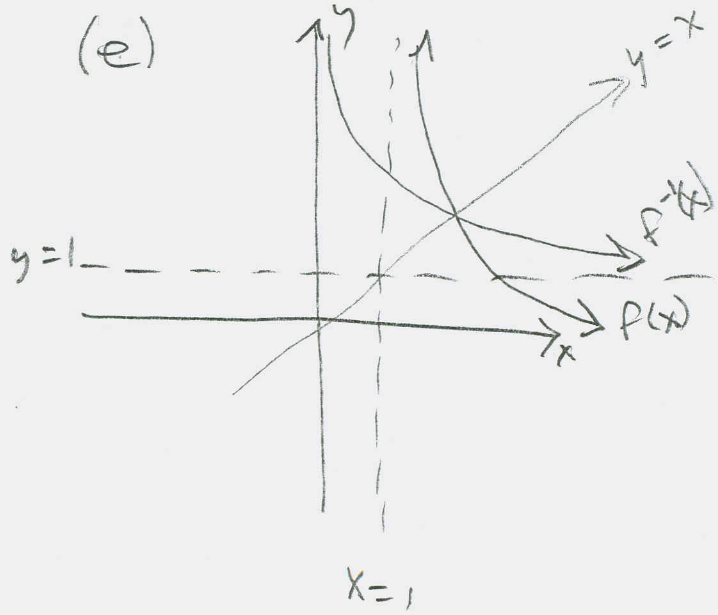
$y = \boxed{\frac{x+1}{x} = f^{-1}(x)}$

(d) $(f^{-1})'(x) = \frac{(1)(x)-(x+1)(1)}{x^2}$

$= \frac{x-x-1}{x^2} = -\frac{1}{x^2} \Longrightarrow$

$(f^{-1})'(2) = -\frac{1}{2^2} = -\frac{1}{4}$ ✓

(e)



202 § 7.1 #s 37, 40, 42

#s 37-40 Find $(f^{-1})'(a)$

37 $f(x) = 2x^3 + 3x^2 + 7x + 4, \quad a = 4$

Check: $f'(x) = 6x^2 + 6x + 7$

$$= 6(x^2 + x \quad ) + 7$$

$$= 6(x^2 + x + (\tfrac{1}{2})^2) + 7 - 6(\tfrac{1}{4})$$

$$= 6(x + \tfrac{1}{2})^2 + 7 - \tfrac{3}{2}$$

$$= 6(x + \tfrac{1}{2})^2 + \tfrac{11}{2} > 0 \ \forall x \Rightarrow \text{strictly increasing}$$

$\Rightarrow$ 1-to-1 on its domain

$f(x) = 4 \Rightarrow x = 0$, by inspection, so $(0,4)$ is the pair for $f(x)$ & so $f^{-1}(4) = 0$.

$$\Rightarrow (f^{-1})'(a) = (f^{-1})'(4) = \frac{1}{f'(f^{-1}(4))} = \frac{1}{f'(0)}$$

$$= \boxed{\frac{1}{7}}$$

40 $f(x) = \sqrt{x^3 + x^2 + x + 1}, \quad a = 2$

$f(x) = 2 \Rightarrow x = 1 = f^{-1}(2)$

$f'(x) = \frac{1}{2}(x^3 + x^2 + x + 1)^{-\frac{1}{2}}(3x^2 + 2x + 1) \Rightarrow$

$f'(f^{-1}(2)) = f'(1) = \frac{1}{2}(1^3 + 1^2 + 1 + 1)^{-\frac{1}{2}}(3(1)^2 + 2(1) + 1)$

$$= \frac{1}{2}(4)^{-\frac{1}{2}}(6) = \frac{1}{4} \cdot 6 = \boxed{\frac{3}{2} = (f^{-1})'(2)}$$

202 § 7.1 #42

42) $\mathcal{S}$ $f$ is difb$^{\ell}$ with inverse $f^{-1}$. Given $G(x) = \dfrac{1}{f^{-1}(x)}$, $f(3)=2$, and $f'(3) = \frac{1}{9}$.

Find $G'(2)$.

we have $f^{-1}(2) = 3$.

$$G'(x) = \frac{d}{dx}\left[\left(f^{-1}(x)\right)^{-1}\right] = -\left(f^{-1}(x)\right)^{-2}\left(f^{-1}\right)'(x)$$

$$= -\left(f^{-1}(x)\right)^{-2} \cdot \frac{1}{f'(f^{-1}(x))} \Longrightarrow$$

$$G'(2) = -\left(f^{-1}(2)\right)^{-2} \cdot \frac{1}{f'(3)} = -(3)^{-2} \cdot \frac{1}{\frac{1}{9}} = -\frac{1}{9} \cdot 9$$

$$= \boxed{-1 = G'(2)}$$