1. We plot T_0, T_2, T_4, and T_6 on the same graph as $\cos(x)$.

$with(plots):$

$f0 := x \to \sum_{k=0}^{0} \frac{(-1)^k x^{2 \cdot k}}{(2 \cdot k)!}$

$$x \to \sum_{k=0}^{0} \frac{(-1)^k x^{2k}}{(2k)!}$$

$f2 := x \to \sum_{k=0}^{1} \frac{(-1)^k x^{2 \cdot k}}{(2 \cdot k)!}$

$$x \to \sum_{k=0}^{1} \frac{(-1)^k x^{2k}}{(2k)!}$$

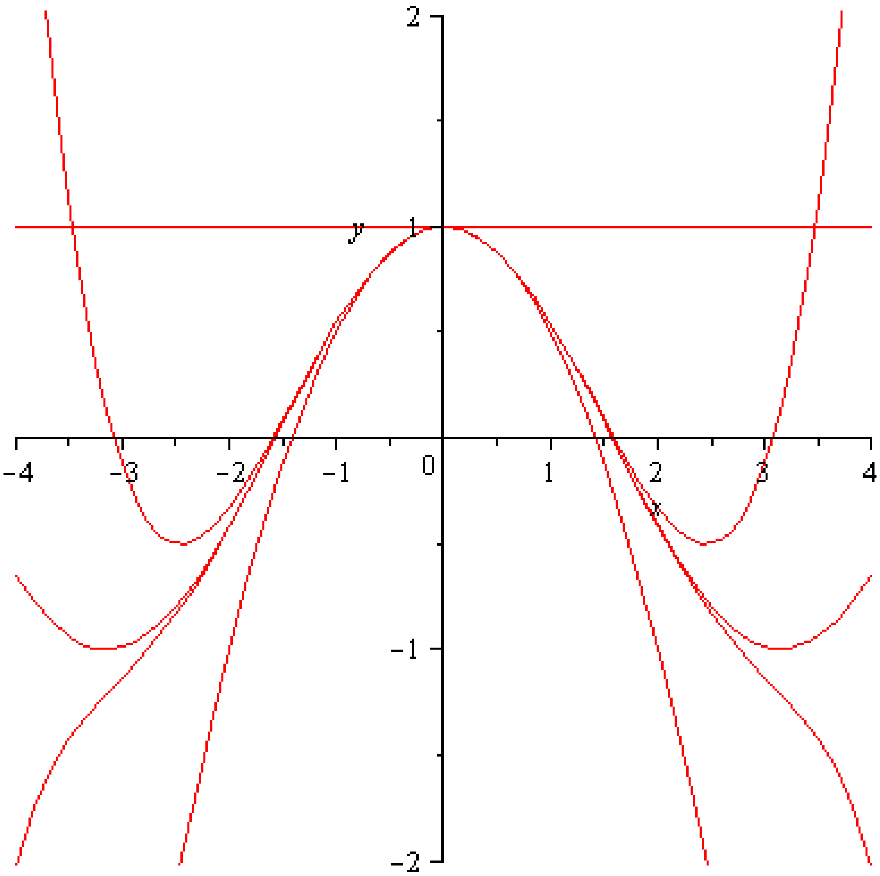
$f4 := x \to \sum_{k=0}^{2} \frac{(-1)^k x^{2 \cdot k}}{(2 \cdot k)!}$

$$x \to \sum_{k=0}^{2} \frac{(-1)^k x^{2k}}{(2k)!}$$

$f6 := x \to \sum_{k=0}^{3} \frac{(-1)^k x^{2 \cdot k}}{(2 \cdot k)!}$

$$x \to \sum_{k=0}^{3} \frac{(-1)^k x^{2k}}{(2k)!}$$

$f0p := plot(f0(x), x = -4..4, y = -2..2) : \% :$
$f2p := plot(f2(x), x = -4..4, y = -2..2) : \% :$
$f4p := plot(f4(x), x = -4..4, y = -2..2) : \% :$
$f6p := plot(f6(x), x = -4..4, y = -2..2) : \% :$
$cosinep := plot(\cos(x), x = -4..4, y = -2..2) : \% :$
$display([f0p, f2p, f4p, f6p, cosinep])$



2
1
y
0
−1
−2
−4
−3
−2
−1
1
2
3
4
x

#s 3 - 10: Find the Taylor polynomial $T_n(x)$ for the function f at the number a. Graph f and T_3 on the same graph.

5. $f(x) = \cos(x),\ a = \dfrac{\pi}{2}$

$f := x \rightarrow \cos(x)$

$$x \rightarrow \cos(x)$$

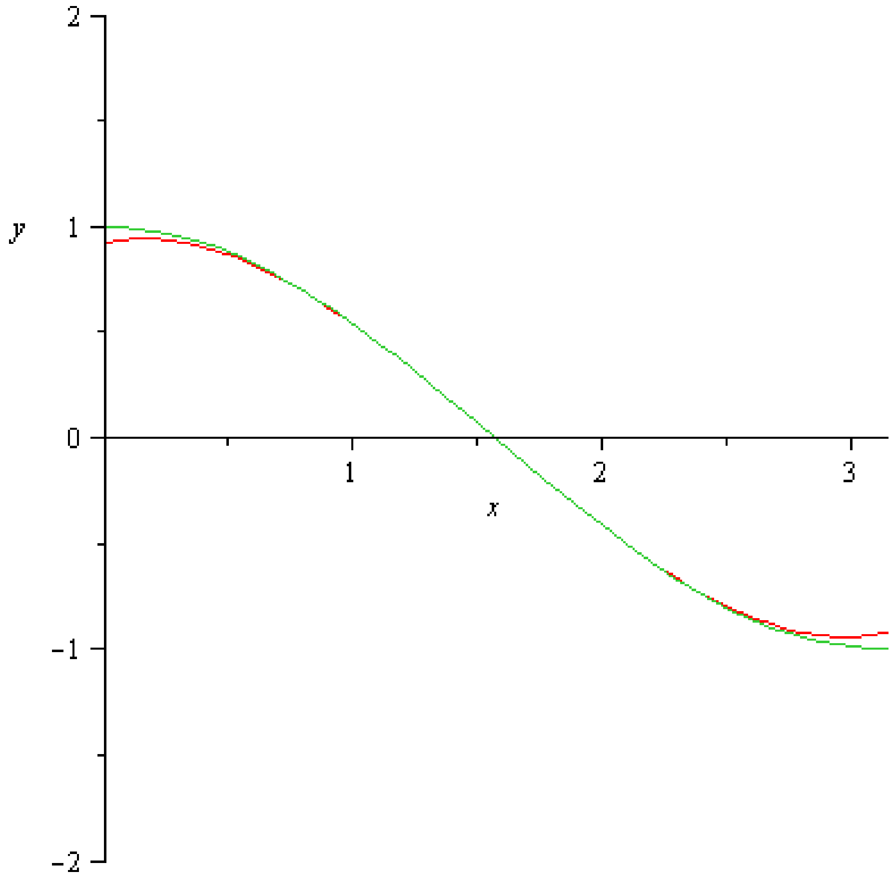
$t3 := taylor\left(f(x), x = \frac{\text{Pi}}{2}, 4\right)$

$$-\left(x - \frac{1}{2}\pi\right) + \frac{1}{6}\left(x - \frac{1}{2}\pi\right)^3 + \mathrm{O}\left(\left(x - \frac{1}{2}\pi\right)^4\right)$$

$t3 := x \rightarrow -\left(x - \frac{1}{2}\pi\right) + \frac{1}{6}\left(x - \frac{1}{2}\pi\right)^3$

$$x \rightarrow -x + \frac{1}{2}\pi + \frac{1}{6}\left(x - \frac{1}{2}\pi\right)^3$$

$plot([t3(x), f(x)], x = 0 .. \text{Pi}, y = -2 .. 2)$



Very good match for values of *x* close to *x* = 0.

8. $f(x) = \frac{\ln(x)}{x}$

8. $f(x) = \frac{\ln(x)}{x}$, $a = 1$

$f := x \rightarrow \frac{\ln(x)}{x}$

$$x \rightarrow \frac{\ln(x)}{x}$$

$f1 := \mathrm{D}(f)$

$$x \rightarrow \frac{1}{x^2} - \frac{\ln(x)}{x^2}$$

$f2 := \mathrm{D}(f1)$

$$x \rightarrow -\frac{3}{x^3} + \frac{2\ln(x)}{x^3}$$

$f3 := \mathrm{D}(f2)$

$$x \rightarrow \frac{11}{x^4} - \frac{6\ln(x)}{x^4}$$

$f4 := \mathrm{D}(f3)$

$$x \rightarrow -\frac{50}{x^5} + \frac{24\ln(x)}{x^5}$$

$f(1)$

$$0$$

$f1(1)$

$$1$$

$f2(1)$

$$-3$$

$f3(1)$

$$11$$

$f4(1)$

$$-50$$

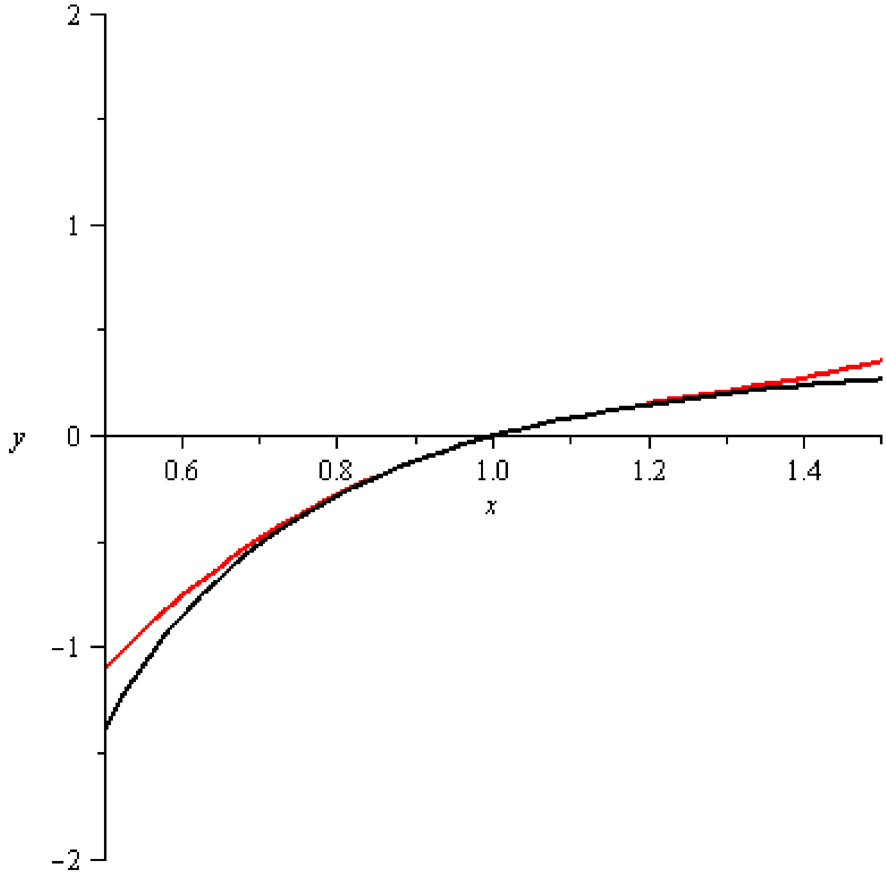
$T_3 := x \rightarrow 1 \cdot (x-1) - \frac{3}{2!} \cdot (x-1)^2 + \frac{11}{3!} \cdot (x-1)^3$

$$x \rightarrow x - 1 - \frac{3(x-1)^2}{2!} + \frac{11(x-1)^3}{3!}$$

$T3plot := plot\left(T_3(x), x = \frac{1}{2} .. \frac{3}{2}, y = -2 .. 2, color = red, thickness = 2\right) : \% :$

$fplot := plot\left(f(x), x = \frac{1}{2} .. \frac{3}{2}, y = -2 .. 2, color = black, thickness = 2\right) : \% :$

$display([T3plot, fplot])$



#s 11, 12: Use a computer algebra system to find the Taylor polynomials T_n *centered at a* **for** $n = 2, 3, 4, 5$. Then graph these polynomials and f on the same screen. I use a souped-up approach using the *taylor* command.

11. $f(x) = \cot(x),\ a = \dfrac{\pi}{4}$

$f := x \rightarrow \cot(x)$

$$x \rightarrow \cot(x)$$

$t2 := x \rightarrow taylor\left(f(x), x = \frac{\text{Pi}}{4}, 3\right)$

$$x \rightarrow taylor\left(f(x), x = \frac{1}{4}\pi, 3\right)$$

$t3 := x \rightarrow taylor\left(f(x), x = \frac{\text{Pi}}{4}, 4\right)$

$$x \rightarrow taylor\left(f(x), x = \frac{1}{4}\pi, 4\right)$$

$t4 := x \rightarrow taylor\left(f(x), x = \frac{\text{Pi}}{4}, 5\right)$

$$x \rightarrow taylor\left(f(x), x = \frac{1}{4}\pi, 5\right)$$

$t5 := x \rightarrow taylor\left(f(x), x = \frac{\text{Pi}}{4}, 6\right)$

$$x \rightarrow taylor\left(f(x), x = \frac{1}{4}\pi, 6\right)$$

Procedure to build the following polynomials: t3(x) enter, copy and paste the 3rd degree polynomial after typing $t3 := x \rightarrow$.

$t2 := x \rightarrow 1 - 2\left(x - \frac{1}{4}\pi\right) + 2\left(x - \frac{1}{4}\pi\right)^2$

$$x \rightarrow 1 - 2x + \frac{1}{2}\pi + 2\left(x - \frac{1}{4}\pi\right)^2$$

$t3 := x \rightarrow 1 - 2\left(x - \frac{1}{4}\pi\right) + 2\left(x - \frac{1}{4}\pi\right)^2 - \frac{8}{3}\left(x - \frac{1}{4}\pi\right)^3$

$$x \rightarrow 1 - 2x + \frac{1}{2}\pi + 2\left(x - \frac{1}{4}\pi\right)^2 - \frac{8}{3}\left(x - \frac{1}{4}\pi\right)^3$$

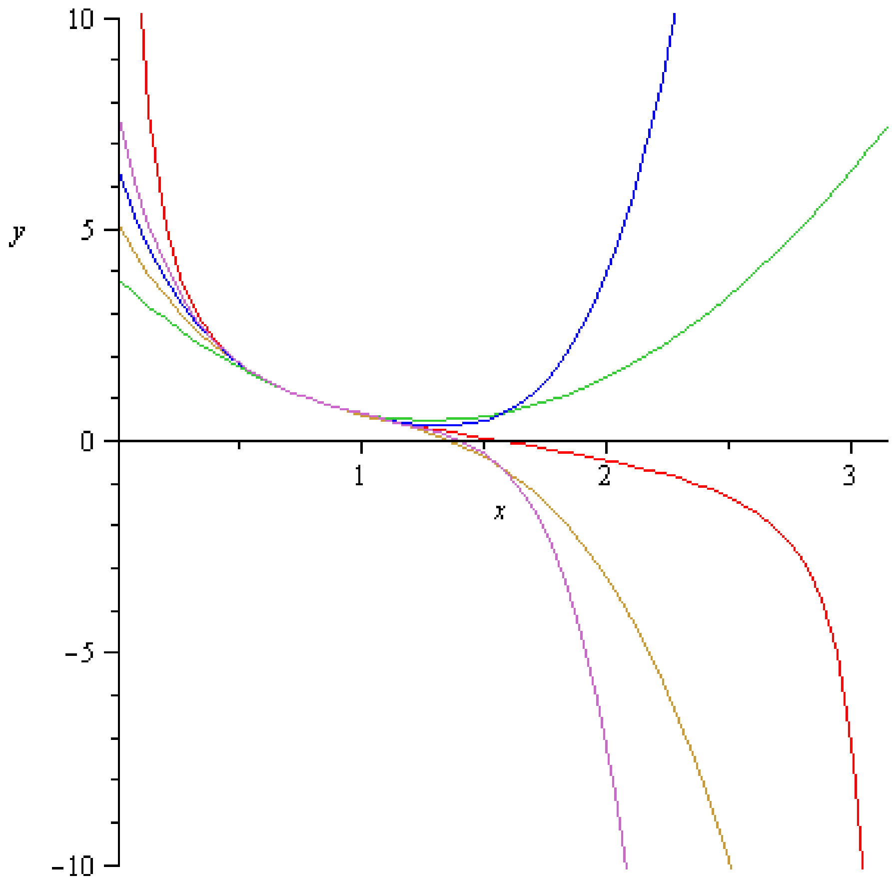
$t4 := x \rightarrow 1 - 2\left(x - \frac{1}{4}\pi\right) + 2\left(x - \frac{1}{4}\pi\right)^2 - \frac{8}{3}\left(x - \frac{1}{4}\pi\right)^3 + \frac{10}{3}\left(x - \frac{1}{4}\pi\right)^4$

$$x \to 1 - 2x + \frac{1}{2}\pi + 2\left(x - \frac{1}{4}\pi\right)^2 - \frac{8}{3}\left(x - \frac{1}{4}\pi\right)^3 + \frac{10}{3}\left(x - \frac{1}{4}\pi\right)^4$$

$$t5 := x \to 1 - 2\left(x - \frac{1}{4}\pi\right) + 2\left(x - \frac{1}{4}\pi\right)^2 - \frac{8}{3}\left(x - \frac{1}{4}\pi\right)^3 + \frac{10}{3}\left(x - \frac{1}{4}\pi\right)^4 - \frac{64}{15}\left(x - \frac{1}{4}\pi\right)^5$$

$$x \to 1 - 2x + \frac{1}{2}\pi + 2\left(x - \frac{1}{4}\pi\right)^2 - \frac{8}{3}\left(x - \frac{1}{4}\pi\right)^3 + \frac{10}{3}\left(x - \frac{1}{4}\pi\right)^4 - \frac{64}{15}\left(x - \frac{1}{4}\pi\right)^5$$

$plot([\cot(x), t2(x), t3(x), t4(x), t5(x)], x = 0..\text{Pi}, y = -10..10)$



12. $f(x) = \sqrt[3]{1 + x^2}$, $a = 0$.

$f := x \to \sqrt[3]{1 + x^2}$

$$x \to (1 + x^2)^{1/3}$$

$t2 := x \to taylor(f(x), x = 0, 3)$

$$x \to taylor(f(x), x = 0, 3)$$

$t3 := x \to taylor(f(x), x = 0, 4)$

$$x \to taylor(f(x), x = 0, 4)$$

$t4 := x \to taylor(f(x), x = 0, 5)$

$$x \to taylor(f(x), x = 0, 5)$$

$t5 := x \to taylor(f(x), x = 0, 6)$

$$x \to taylor(f(x), x = 0, 6)$$

$t2 := x \to 1 + \frac{1}{3} x^2$

$$x \to 1 + \frac{1}{3} x^2$$

$t3 := x \to 1 + \frac{1}{3} x^2$

$$x \to 1 + \frac{1}{3} x^2$$

$t4 := x \to 1 + \frac{1}{3} x^2 - \frac{1}{9} x^4$

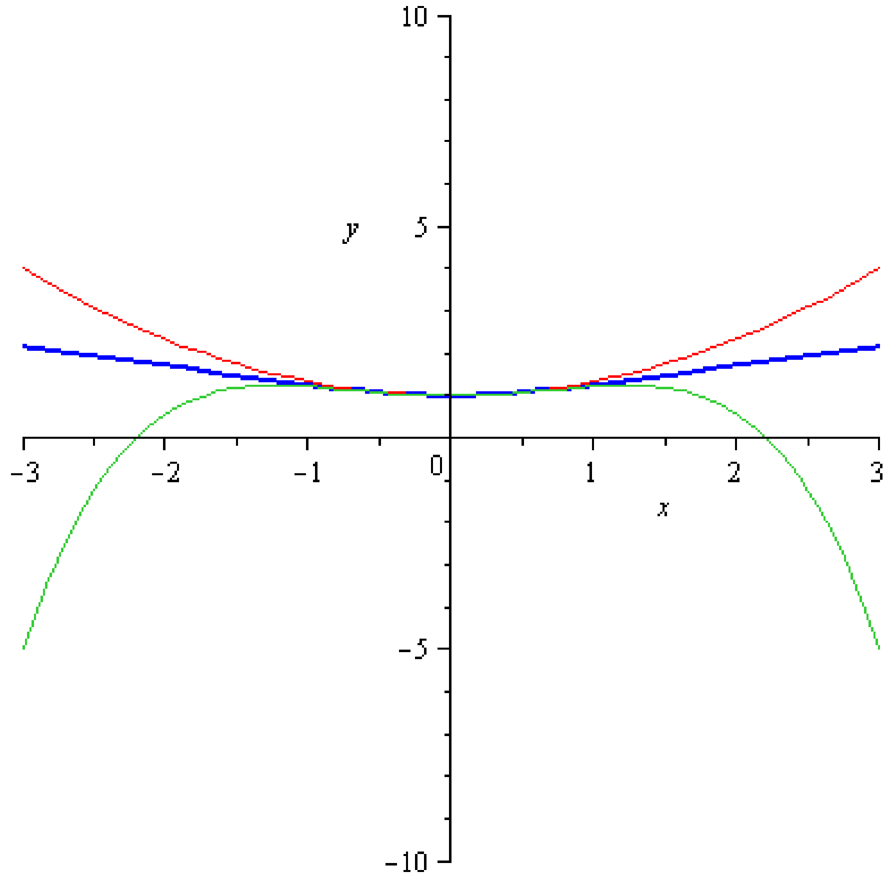
$$x \to 1 + \frac{1}{3} x^2 - \frac{1}{9} x^4$$

$t3, t5$ don't change anything.

$tplots := plot([t2(x), t4(x)], x = -3..3, y = -10..10) : \% :$

$fplot := plot(f(x), x = -3..3, y = -10..10, color = blue, thickness = 2) : \% :$

$display([fplot, tplots])$



10
5
y
-3
-2
-1
0
1
2
3
x
-5
-10

#s 13 - 22

(a) Approximate *f* by a Taylor plynomial with degree *n* at the number *a*.

(b) Use Taylor's Inequality to estimate the accuracy of the approximation, when *x* lies in the given interval.

(c) Check your result in part (b) by graphing $|R_n(x)|$.

16. $f(x) = \sin(x),\ a = \dfrac{\pi}{6},\ n = 2,\ 4 \le x \le 4.2$

$f := x \rightarrow \sin(x)$

$$x \rightarrow \sin(x)$$

$t := x \rightarrow taylor\left(f(x), x = \frac{\mathrm{Pi}}{6}, 3\right)$

$$x \rightarrow taylor\left(f(x), x = \frac{1}{6}\pi, 3\right)$$

$t(x)$

$$\frac{1}{2} + \frac{1}{2}\sqrt{3}\left(x - \frac{1}{6}\pi\right) - \frac{1}{4}\left(x - \frac{1}{6}\pi\right)^2 + \mathrm{O}\left(\left(x - \frac{1}{6}\pi\right)^3\right)$$

$t2 := x \rightarrow \frac{1}{2} + \frac{1}{2}\sqrt{3}\left(x - \frac{1}{6}\pi\right) - \frac{1}{4}\left(x - \frac{1}{6}\pi\right)^2$

$$x \rightarrow \frac{1}{2} + \frac{1}{2}\sqrt{3}\left(x - \frac{1}{6}\pi\right) - \frac{1}{4}\left(x - \frac{1}{6}\pi\right)^2$$

21. $f(x) = x \cdot \sin(x),\ a = 0,\ n = 4, -1 \le x \le 1$

$f := x \to x \cdot \sin(x)$

$$x \to x \sin(x)$$

$t := x \to taylor(f(x), x = 0, 5)$

$$x \to taylor(f(x), x = 0, 5)$$

$t(x)$

$$x^2 - \frac{1}{6} x^4 + O(x^6)$$

$t4 := x \to x^2 - \frac{1}{6} x^4$

$$x \to x^2 - \frac{1}{6} x^4$$

23.

23. Use #5 to approximate $\cos(80^0)$ correct to five decimal places. Have to execute the #5 stuff to get *f* and *t3* acting correctly.

$t3(x)$

$$-x + \frac{1}{2}\pi + \frac{1}{6}\left(x - \frac{1}{2}\pi\right)^3$$

$t3\left(\frac{80 \cdot \mathrm{Pi}}{180}\right)$

$$\frac{1}{18}\pi - \frac{1}{34992}\pi^3$$

$evalf(\%)$

$$0.173646829$$

So it looks like **0.17365** is correct to 5 places. I'm going to double check by looking at the next term in this alternating series:

$taylor\left(\cos(x), x = \frac{Pi}{2}, 6\right)$

$$-\left(x - \frac{1}{2}\pi\right) + \frac{1}{6}\left(x - \frac{1}{2}\pi\right)^3 - \frac{1}{120}\left(x - \frac{1}{2}\pi\right)^5 + O\left(\left(x - \frac{1}{2}\pi\right)^6\right)$$

$t5 := x \rightarrow -\left(x - \frac{1}{2}\pi\right) + \frac{1}{6}\left(x - \frac{1}{2}\pi\right)^3 - \frac{1}{120}\left(x - \frac{1}{2}\pi\right)^5$

$$x \rightarrow -x + \frac{1}{2}\pi + \frac{1}{6}\left(x - \frac{1}{2}\pi\right)^3 - \frac{1}{120}\left(x - \frac{1}{2}\pi\right)^5$$

$-\frac{1}{120}\left(\frac{80 \cdot Pi}{180} - \frac{1}{2}\pi\right)^5$

$$\frac{1}{226748160}\pi^5$$

$evalf(\%)$

$$0.00000134960162$$

0.000001349601624+ 0.173646829(

$$0.173648178($$

Sure enough, it doesn't change in the 5th digit, when I add the next term in there.   Good.

24.

Use info from #16 to evaluate $\sin(38^0)$ correct to five decimal places.

$t2 := x \rightarrow \frac{1}{2} + \frac{1}{2}\sqrt{3}\left(x - \frac{1}{6}\pi\right) - \frac{1}{4}\left(x - \frac{1}{6}\pi\right)^2$

$$x \rightarrow \frac{1}{2} + \frac{1}{2}\sqrt{3}\left(x - \frac{1}{6}\pi\right) - \frac{1}{4}\left(x - \frac{1}{6}\pi\right)^2$$

$t2\left(\frac{38 \cdot Pi}{180}\right)$

$$\frac{1}{2} + \frac{1}{45}\sqrt{3}\,\pi - \frac{1}{2025}\pi^2$$

$evalf(\%)$

$$0.616046079($$

From this, it appears that we have   **0.61605** gives us the desired result.

25.

Use Taylor's Inequality **to** *determine the n*