2. The series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ has positive terms and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \left[\frac{(n+1)^2}{2^{n+1}} \cdot \frac{2^n}{n^2}\right] = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^2 \cdot \frac{1}{2} = \frac{1}{2} < 1$, so the series is absolutely convergent by the Ratio Test.

3. $\sum_{n=0}^{\infty} \frac{(-10)^n}{n!}$. Using the Ratio Test, $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(-10)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-10)^n}\right| = \lim_{n\to\infty} \left|\frac{-10}{n+1}\right| = 0 < 1$, so the series is absolutely convergent.

4. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^n}{n^4}$ diverges by the Test for Divergence. $\lim_{n\to\infty} \frac{2^n}{n^4} = \infty$, so $\lim_{n\to\infty} (-1)^{n-1} \frac{2^n}{n^4}$ does not exist.

5. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[4]{n}}$ converges by the Alternating Series Test, but $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}}$ is a divergent p-series $\left(p = \frac{1}{4} \le 1\right)$, so the given series is conditionally convergent.

8. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left[\frac{(n+1)!}{100^{n+1}} \cdot \frac{100^n}{n!}\right] = \lim_{n\to\infty} \frac{n+1}{100} = \infty$, so the series $\sum_{n=1}^{\infty} \frac{n!}{100^n}$ diverges by the Ratio Test.

10. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+2}}$ converges by the Alternating Series Test (see Exercise 11.5.8). Let $a_n = \frac{1}{\sqrt{n}}$ with $b_n = \frac{n}{\sqrt{n^3+2}}$.

Then $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \left(\frac{1}{\sqrt{n}} \cdot \frac{\sqrt{n^3+2}}{n}\right) = \lim_{n\to\infty} \frac{\sqrt{n^3+2}}{\sqrt{n^3}} = \lim_{n\to\infty} \sqrt{1+\frac{2}{n^3}} = 1 > 0$, so $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+2}}$ diverges by limit comparision with the divergent p-series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ $\left[p = \frac{1}{2} \le 1\right]$. Thus, $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+2}}$ is conditionally convergent.

12. $\left|\frac{\sin 4n}{4^n}\right| \le \frac{1}{4^n}$, so $\sum_{n=1}^{\infty} \left|\frac{\sin 4n}{4^n}\right|$ converges by comparison with the convergent geometric series $\sum_{n=1}^{\infty} \frac{1}{4^n}$ $\left[|r| = \frac{1}{4} < 1\right]$.

Thus, $\sum_{n=1}^{\infty} \frac{\sin 4n}{4^n}$ is absolutely convergent.

13. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left[\frac{10^{n+1}}{(n+2)\,4^{2n+3}} \cdot \frac{(n+1)\,4^{2n+1}}{10^n}\right] = \lim_{n\to\infty} \left(\frac{10}{4^2} \cdot \frac{n+1}{n+2}\right) = \frac{5}{8} < 1$, so the series $\sum_{n=1}^{\infty} \frac{10^n}{(n+1)4^{2n+1}}$ is absolutely convergent by the Ratio Test. Since the terms of this series are positive, absolute convergence is the same as convergence.

18. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+1)!/\,(n+1)^{n+1}}{n!/n^n} = \lim_{n\to\infty} \frac{n^n}{(n+1)^n} = \lim_{n\to\infty} \frac{1}{(1+1/n)^n} = \frac{1}{e} < 1$, so the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converges absolutely by the Ratio Test.

20. $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left|\frac{(-2)^n}{n^n}\right|} = \lim_{n\to\infty} \frac{2}{n} = 0 < 1$, so the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$ is absolutely convergent by the Root Test.

23. $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left(1+\frac{1}{n}\right)^{n^2}} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e > 1$ (by Equation 3.6.6), so the series $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{n^2}$ diverges by the Root Test.

25. Use the Ratio Test with the series

$$1 - \frac{1\cdot 3}{3!} + \frac{1\cdot 3\cdot 5}{5!} - \frac{1\cdot 3\cdot 5\cdot 7}{7!} + \cdots + (-1)^{n-1}\frac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}{(2n-1)!} + \cdots = \sum_{n=1}^{\infty} (-1)^{n-1}\frac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}{(2n-1)!}.$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^n\cdot 1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)[2(n+1)-1]}{[2(n+1)-1]!}\cdot\frac{(2n-1)!}{(-1)^{n-1}\cdot 1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(-1)(2n+1)(2n-1)!}{(2n+1)(2n)(2n-1)!}\right| = \lim_{n\to\infty}\frac{1}{2n} = 0 < 1,$$

so the given series is absolutely convergent and therefore convergent.

27. $\sum_{n=1}^{\infty} \frac{2\cdot 4\cdot 6\cdot\cdots\cdot(2n)}{n!} = \sum_{n=1}^{\infty} \frac{(2\cdot 1)\cdot(2\cdot 2)\cdot(2\cdot 3)\cdot\cdots\cdot(2\cdot n)}{n!} = \sum_{n=1}^{\infty} \frac{2^n n!}{n!} = \sum_{n=1}^{\infty} 2^n$, which diverges by the Test for Divergence since $\lim_{n\to\infty} 2^n = \infty$.

30. By the recursive definition, $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{2+\cos n}{\sqrt{n}}\right| = 0 < 1$, so the series converges absolutely by the Ratio Test.

35. (a) $s_5 = \sum_{n=1}^{5} \frac{1}{n2^n} = \frac{1}{2} + \frac{1}{8} + \frac{1}{24} + \frac{1}{64} + \frac{1}{160} = \frac{661}{960} \approx 0.68854$. Now the ratios

$r_n = \frac{a_{n+1}}{a_n} = \frac{n2^n}{(n+1)2^{n+1}} = \frac{n}{2(n+1)}$ form an increasing sequence, since

$r_{n+1} - r_n = \frac{n+1}{2(n+2)} - \frac{n}{2(n+1)} = \frac{(n+1)^2 - n(n+2)}{2(n+1)(n+2)} = \frac{1}{2(n+1)(n+2)} > 0$. So by Exercise 34(b), the error

in using s_5 is $R_5 \le \frac{a_6}{1 - \lim_{n\to\infty} r_n} = \frac{1/(6\cdot 2^6)}{1 - 1/2} = \frac{1}{192} \approx 0.00521$.

(b) The error in using s_n as an approximation to the sum is $R_n = \frac{a_{n+1}}{1-\frac{1}{2}} = \frac{2}{(n+1)2^{n+1}}$. We want $R_n < 0.00005 \quad\Leftrightarrow$

$\frac{1}{(n+1)2^n} < 0.00005 \quad\Leftrightarrow\quad (n+1)2^n > 20{,}000$. To find such an n we can use trial and error or a graph. We calculate

$(11+1)2^{11} = 24{,}576$, so $s_{11} = \sum_{n=1}^{11} \frac{1}{n2^n} \approx 0.693109$ is within 0.00005 of the actual sum.

36. $s_{10} = \sum_{n=1}^{10} \frac{n}{2^n} = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \cdots + \frac{10}{1024} \approx 1.988$. The ratios $r_n = \frac{a_{n+1}}{a_n} = \frac{n+1}{2^{n+1}} \cdot \frac{2^n}{n} = \frac{n+1}{2n} = \frac{1}{2}\left(1 + \frac{1}{n}\right)$ form a decreasing sequence, and $r_{11} = \frac{11+1}{2(11)} = \frac{12}{22} = \frac{6}{11} < 1$, so by Exercise 34(a), the error in using s_{10} to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$ is $R_{10} \leq \frac{a_{11}}{1 - r_{11}} = \frac{\frac{11}{2048}}{1 - \frac{6}{11}} = \frac{121}{10{,}240} \approx 0.0118$.