6. $\sum\limits_{n=1}^{\infty} a_n = \sum\limits_{n=1}^{\infty} (-1)^{n-1} \dfrac{1}{\ln(n+4)} = \sum\limits_{n=1}^{\infty} (-1)^{n-1} b_n$. Now $b_n = \dfrac{1}{\ln(n+4)} > 0$, $\{b_n\}$ is decreasing, and $\lim\limits_{n\to\infty} b_n = 0$, so the series converges by the Alternating Series Test.

7. $\sum\limits_{n=1}^{\infty} a_n = \sum\limits_{n=1}^{\infty} (-1)^n \dfrac{3n-1}{2n+1} = \sum\limits_{n=1}^{\infty} (-1)^n b_n$. Now $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \dfrac{3-1/n}{2+1/n} = \dfrac{3}{2} \neq 0$. Since $\lim\limits_{n\to\infty} a_n \neq 0$ (in fact the limit does not exist), the series diverges by the Test for Divergence.

11. $b_n = \dfrac{n^2}{n^3+4} > 0$ for $n \geq 1$. $\{b_n\}$ is decreasing for $n \geq 2$ since

$$\left(\frac{x^2}{x^3+4}\right)' = \frac{(x^3+4)(2x) - x^2(3x^2)}{(x^3+4)^2} = \frac{x(2x^3+8-3x^3)}{(x^3+4)^2} = \frac{x(8-x^3)}{(x^3+4)^2} < 0 \text{ for } x > 2. \text{ Also,}$$

$\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \dfrac{1/n}{1+4/n^3} = 0$. Thus, the series $\sum\limits_{n=1}^{\infty} (-1)^{n+1} \dfrac{n^2}{n^3+4}$ converges by the Alternating Series Test.

12. $b_n = \dfrac{e^{1/n}}{n} > 0$ for $n \geq 1$. $\{b_n\}$ is decreasing since $\left(\dfrac{e^{1/x}}{x}\right)' = \dfrac{x \cdot e^{1/x}(-1/x^2) - e^{1/x} \cdot 1}{x^2} = \dfrac{-e^{1/x}(1+x)}{x^3} < 0$ for $x > 0$. Also, $\lim\limits_{n\to\infty} b_n = 0$ since $\lim\limits_{n\to\infty} e^{1/n} = 1$. Thus, the series $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \dfrac{e^{1/n}}{n}$ converges by the Alternating Series Test.

13. $\sum\limits_{n=2}^{\infty} (-1)^n \dfrac{n}{\ln n}$. $\lim\limits_{n\to\infty} \dfrac{n}{\ln n} = \lim\limits_{x\to\infty} \dfrac{x}{\ln x} \overset{H}{=} \lim\limits_{x\to\infty} \dfrac{1}{1/x} = \infty$, so the series diverges by the Test for Divergence.

14. $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \left(\dfrac{\ln n}{n}\right) = 0 + \sum\limits_{n=2}^{\infty} (-1)^{n-1} \left(\dfrac{\ln n}{n}\right)$. $b_n = \dfrac{\ln n}{n} > 0$ for $n \geq 2$, and if $f(x) = \dfrac{\ln x}{x}$, then $f'(x) = \dfrac{1-\ln x}{x^2} < 0$ for $x > e$, so $\{b_n\}$ is eventually decreasing. Also,

$\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \dfrac{\ln n}{n} = \lim\limits_{x\to\infty} \dfrac{\ln x}{x} \overset{H}{=} \lim\limits_{x\to\infty} \dfrac{1/x}{1} = 0$, so the series converges by the Alternating Series Test.

15. $\sum\limits_{n=1}^{\infty} \dfrac{\cos n\pi}{n^{3/4}} = \sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{n^{3/4}}$. $b_n = \dfrac{1}{n^{3/4}}$ is decreasing and positive and $\lim\limits_{n\to\infty} \dfrac{1}{n^{3/4}} = 0$, so the series converges by the Alternating Series Test.

16. $\sin\left(\dfrac{n\pi}{2}\right) = 0$ if n is even and $(-1)^k$ if $n = 2k+1$, so the series $\sum\limits_{n=1}^{\infty} \dfrac{\sin(n\pi/2)}{n!} = \sum\limits_{n=0}^{\infty} \dfrac{(-1)^n}{(2n+1)!}$.

$b_n = \dfrac{1}{(2n+1)!} > 0$, $\{b_n\}$ is decreasing, and $\lim\limits_{n\to\infty} \dfrac{1}{(2n+1)!} = 0$, so the series converges by the Alternating Series Test.

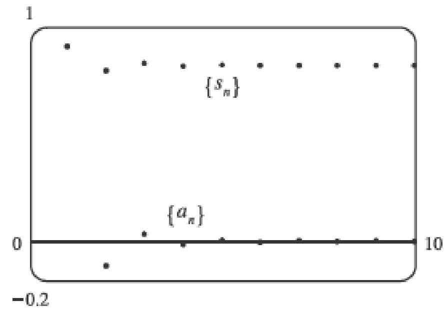
17. $\sum\limits_{n=1}^{\infty} (-1)^n \sin\left(\dfrac{\pi}{n}\right)$. $b_n = \sin\left(\dfrac{\pi}{n}\right) > 0$ for $n \geq 2$ and $\sin\left(\dfrac{\pi}{n}\right) \geq \sin\left(\dfrac{\pi}{n+1}\right)$, and $\lim\limits_{n\to\infty} \sin\left(\dfrac{\pi}{n}\right) = \sin 0 = 0$, so the series converges by the Alternating Series Test.

18. $\sum_{n=1}^{\infty}(-1)^n\cos\left(\frac{\pi}{n}\right)$. $\lim_{n\to\infty}\cos\left(\frac{\pi}{n}\right)=\cos(0)=1$, so $\lim_{n\to\infty}(-1)^n\cos\left(\frac{\pi}{n}\right)$ does not exist and the series diverges by the Test for Divergence.

19. $\frac{n^n}{n!}=\frac{n\cdot n\cdot\cdots\cdot n}{1\cdot 2\cdot\cdots\cdot n}\geq n \quad\Rightarrow\quad \lim_{n\to\infty}\frac{n^n}{n!}=\infty \quad\Rightarrow\quad \lim_{n\to\infty}\frac{(-1)^n\,n^n}{n!}$ does not exist. So the series diverges by the Test for Divergence.

22.

n	a_n	s_n
1	1	1
2	−0.125	0.875
3	0.03704	0.91204
4	−0.01563	0.89641
5	0.008	0.90441
6	−0.00463	0.89978
7	0.00292	0.90270
8	−0.00195	0.90074
9	0.00137	0.90212
10	−0.001	0.90112



By the Alternating Series Estimation Theorem, the error in the approximation $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^3}\approx 0.90112$ is $|s-s_{10}|\leq b_{11}=1/11^3\approx 0.0007513$.

24. The series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n\,5^n}$ satisfies (i) of the Alternating Series Test because $\frac{1}{(n+1)5^{n+1}}<\frac{1}{n\,5^n}$ and (ii) $\lim_{n\to\infty}\frac{1}{n\,5^n}=0$, so the series is convergent. Now $b_4=\frac{1}{4\cdot 5^4}=0.0004>0.0001$ and $b_5=\frac{1}{5\cdot 5^5}=0.000064<0.0001$, so by the Alternating Series Estimation Theorem, $n=4$. (That is, since the 5th term is less than the desired error, we need to add the first 4 terms to get the sum to the desired accuracy.)

29. $b_7=\frac{7^2}{10^7}=0.000\,004\,9$, so

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}n^2}{10^n}\approx s_6=\sum_{n=1}^{6}\frac{(-1)^{n-1}n^2}{10^n}=\frac{1}{10}-\frac{4}{100}+\frac{9}{1000}-\frac{16}{10{,}000}+\frac{25}{100{,}000}-\frac{36}{1{,}000{,}000}=0.067\,614.$$

Adding b_7 to s_6 does not change the fourth decimal place of s_6, so the sum of the series, correct to four decimal places, is 0.0676.

31. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{49}-\frac{1}{50}+\frac{1}{51}-\frac{1}{52}+\cdots$. The 50th partial sum of this series is an underestimate, since $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}=s_{50}+\left(\frac{1}{51}-\frac{1}{52}\right)+\left(\frac{1}{53}-\frac{1}{54}\right)+\cdots$, and the terms in parentheses are all positive. The result can be seen geometrically in Figure 1.

32. If $p > 0$, $\dfrac{1}{(n+1)^p} \le \dfrac{1}{n^p}$ ($\{1/n^p\}$ is decreasing) and $\lim\limits_{n\to\infty} \dfrac{1}{n^p} = 0$, so the series converges by the Alternating Series Test.

If $p \le 0$, $\lim\limits_{n\to\infty} \dfrac{(-1)^{n-1}}{n^p}$ does not exist, so the series diverges by the Test for Divergence. Thus, $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n-1}}{n^p}$ converges $\Leftrightarrow$ $p > 0$.

33. Clearly $b_n = \dfrac{1}{n+p}$ is decreasing and eventually positive and $\lim\limits_{n\to\infty} b_n = 0$ for any p. So the series converges (by the Alternating Series Test) for any p for which every b_n is defined, that is, $n + p \ne 0$ for $n \ge 1$, or p is not a negative integer.

34. Let $f(x) = \dfrac{(\ln x)^p}{x}$. Then $f'(x) = \dfrac{(\ln x)^{p-1}(p - \ln x)}{x^2} < 0$ if $x > e^p$ so f is eventually decreasing for every p. Clearly $\lim\limits_{n\to\infty} \dfrac{(\ln n)^p}{n} = 0$ if $p \le 0$, and if $p > 0$ we can apply l'Hospital's Rule $[\![p+1]\!]$ times to get a limit of 0 as well. So the series converges for all p (by the Alternating Series Test).